NAVAL HEALTH RESEARCH CENTER

INTRINSICALLY LINEAR LOSS DEVELOPMENT MODELS FOR WORKERS' COMPENSATION COSTS: POINT AND INTERVAL PREDICTION METHODS

J. E. Angus

Report No. 94-13

Approved for public release: distribution unlimited.

NAVAL HEALTH RESEARCH CENTER
P. O. BOX 85122
SAN DIEGO, CALIFORNIA 92186 – 5122

NAVAL MEDICAL RESEARCH AND DEVELOPMENT COMMAND
BETHESDA, MARYLAND

INTRINSICALLY LINEAR LOSS DEVELOPMENT MODELS FOR WORKERS' COMPENSATION COSTS: POINT AND INTERVAL PREDICTION METHODS

J. E. ANGUS

Department of Mathematics
The Claremont Graduate School
143 E. Tenth Street
Claremont, CA 91711

and

Naval Health Research Center
Medical Information Systems and
Operations Research Department
P.O. Box 85122
San Diego, California 92186-5122

August 1994

Report 94-13 is supported by the Naval Health Research Center under the American Society for Engineering Education Navy Summer Faculty Research Program. The views expressed in this report are those of the author and do not reflect the official policy or position of the Department of the Navy, Department of Defense, or the U.S. Government Approved for public release, distribution unlimited.

SUMMARY

This report describes modeling methods that allow the computation of point predictions and prediction probability intervals for cumulative workers' compensation costs. Underlying these models is the actuarial loss development factor method, a method that computes projected costs by utilizing ratios of known cumulative costs in consecutive years. While the relationship between cumulative loss development, cohort, and development year in these models is nonlinear, a transformation renders them in the form of standard linear statistical models, thus allowing the development of prediction probability intervals when the error structure is Gaussian. The modeling methods are illustrated using data collected from U.S. Department of the Navy workers' compensation payments made from 1990 through 1993, including claim costs originating from 1961 through 1993.

1. Introduction

The usual and preferred actuarial method for projecting workers' compensation claim costs is based on the so-called Loss Development Model. To illustrate this method, Table 1.1 lists actual total (indemnity and medical combined) cumulative claim costs (in thousands of dollars) from cohorts of claimants originating in years 1990 through 1993.[1]

The column for development year 1 is simply the total claim costs incurred for the year in which the claims originated, namely, the cohort year. Cumulative costs are available for the 1990 cohort for years 1990, 1991, 1992, and 1993. In contrast, data are not available, for example, for the 1992 cohort in development years 3 and 4, since these would be accumulated in the years 1994 and 1995 and are not yet available.

Table 1.1. Actual Total (Indemnity and Medical) Cumulative Claim Costs (In Thousands $)

	Development Year			
Cohort Year	1	2	3	4
1990	14,955	41,424	62,897	79,971
1991	13,566	40,314	60,137	*
1992	14,468	41,892	*	*
1993	13,702	*	*	*

It is of interest, based on the above data, to project (or forecast, predict) the missing costs represented by the asterisks. Actuaries approach this problem by computing Loss Development Factors (LDF) for consecutive years. For a given cohort, these are simply the ratios of the cumulative costs from consecutive years. Actuaries treat LDFs across cohorts as homogeneous and average them with the intention of improving precision. Finally, they compute cumulative LDFs by taking cumulative products of these averages. These computations are summarized for the data in Table 1.2. Notice that by construction, the successive costs for a given cohort may be computed by multiplying the cohort year cost (development year 1) successively by the LDFs. For example, the 1990 cohort generates (in thousands) $14,955 the first year, ($14,955)

(2.76991) = $41,424 accumulated through the second year, ($14,955) (2.76991) (1.51837) = $62,897 accumulated through the third year, and so on.

Table 1.2. Loss Development Factors Computed from Table 1.1.

Cohort Year	Development Year 1 to 2	2 to 3	3 to 4
1990	2.76991	1.51837	1.27146
1991	2.97169	1.49172	-
1992	2.89549	-	-
Average:	2.87903	1.50504	1.27146
Cumulative:	2.87903	4.33307	5.50932

Through experience, actuaries have found that applying the estimated (average) LDFs calculated in this fashion to the cohorts with missing data yields accurate projections. That is, to project the accumulated cost through development year 2 for the 1993 cohort, one simply computes ($13,702) (2.87903) = $39,449. Similarly, the accumulated cost (using cumulative LDFs calculated from the average LDFs) through development year 3 for the 1993 cohort is projected to be ($13,702) (4.33307) = $59,372. Proceeding in this fashion, the missing values in Table 1.1 are projected as shown in Table 1.3.

Table 1.3. Point Predictions of Missing Costs from Table 1.1.

Cohort Year	Development Year 1	2	3	4
1990	-	-	-	-
1991	-	-	-	76,462
1992	-	-	63,049	80,165
1993	-	39,449	59,372	75,489

Actuaries use *numerous variations* of this method. For instance, the method may be applied separately to indemnity and medical costs to achieve greater projection accuracy. Also,

volume-weighted averages may be used instead of simple averages of the columns of LDFs. Finally, the costs entered in the model may or may not be adjusted for economic factors, such as inflation, with the understanding that the resulting predictions would retain the same interpretation with respect to these factors. That is, if the costs entered into the model are (or are not) adjusted for inflation, then the predictions are (or are not) adjusted for inflation. Typically, however, actuaries only present their results as point predictions, and no method for computing measures of prediction accuracy are offered. (Indeed, searches of the literature and consultation with practicing actuaries indicate that such methods have not been widely investigated.)

Many sources of potential error are inherent in the LDF method. Data of this nature contain random fluctuations and other errors (e.g., systematic, specification). Also, when a large number of cohort years are involved, comparable LDFs across cohorts can have a trend. For example, the ratio of development year 2 cumulative cost to development year 1 cost for the 1961 cohort could be significantly larger than that for the 1992 cohort. Having not accounted for this trend, its effects show up as error in the basic LDF model previously described. To develop an assessment of prediction accuracy, the LDF method must be generalized and put into the context of a statistical model that accounts for and makes assumptions concerning the nature of statistical error. The natural way in which to assess prediction error is to accept the prediction itself as a random variable having a certain probability distribution, and to provide not only a numerical value for the prediction, but also an interval with the interpretation that the true value of the cost will fall into that interval with a preselected probability.

Mathematically, the problem may be described as follows. A cost prediction C is a function of available observed data $C_1, C_2, ..., C_n$, say $C = f(C_1, ..., C_n)$. A prediction interval $\left(L(C_1, ..., C_n), U(C_1, ..., C_n)\right)$ for C with coverage probability p is a random interval with the property that $p = P\left\{ L(C_1, ..., C_n) < C < U(C_1, ..., C_n) \right\}$. Naturally, the larger (or smaller) the level of p, the wider (or narrower) the prediction interval will be. For a fixed p, the width of the prediction interval reflects the accuracy of the prediction.

The remainder of this report develops intrinsically linear statistical LDF models from which predictions and prediction intervals may be computed. The basic models are nonlinear,

but are termed "intrinsically linear" because they become standard linear statistical models after a natural-log transformation. This intrinsically linear structure makes the development of prediction probability intervals more tractable. Section 2 develops the mathematical structure of the models, and section 3 applies the models to the computation of predictions and prediction intervals for U.S. Department of the Navy workers' compensation costs.

Two sources of data are used in section 3. Complete data collected by the U.S. Department of the Navy for 1990, 1991, 1992, and 1993 cohorts are used to illustrate one of the models. Yearly incremental costs and claim counts were not available in the U.S. Department of the Navy database for the 1961 through 1988 cohorts for development years prior to 1990 (but are available from 1990 forward). To study the available data and to make future year projections of total cumulative costs Miccolis[3] used actuarial methods to "reverse forecast" the cumulative costs for the 1961 through 1989 cohorts for development years prior to 1990 by examining trends in the incremental cost and claim count data available. These imputed data, along with the actual data, are used by Miccolis[3] to then produce projections of future year cumulative costs. In section 3, these imputed and actual data are used as though they were all actual data to illustrate the use of one of the models developed herein.

The final section contains conclusions of this investigation and recommendations for further study.

2. Intrinsically Linear Statistical Loss Development Models

Throughout this section, the basic data available will be assumed to consist of cumulative costs (accumulated over development years) $C_{i,j}$, $1 \leq i \leq N-j+1$, $1 \leq j \leq N$. Here, the subscript i designates the ith cohort and the subscript j designates the development year. N is the total number of cohorts. Usually, and this will be assumed here, the cohorts are from successive years and arranged in increasing order by year, and assigned values 1 through N, respectively. This facilitates analysis of a trend in LDF across cohorts. In the example in the introduction, N = 4, and the cohorts 1990, 1991, 1992, 1993 are assigned values 1, 2, 3, and 4, respectively. Only the upper left triangular (including the main diagonal–see Table 1.1) area of the cost matrix elements are observed; the object is to predict the cost values in the lower right triangle

(and beyond).

Define $Y_{ij} = \log(C_{i,j}/C_{i,j-1})$, for $1 \le i \le N-j+1$, $2 \le j \le N$. Notice that $\exp(Y_{ij})$ is the LDF relating year j cumulative costs to year j-1 cumulative costs for cohort i. It is postulated that

$$Y_{ij} = f(i,j) + \varepsilon_{ij} \tag{2.1}$$

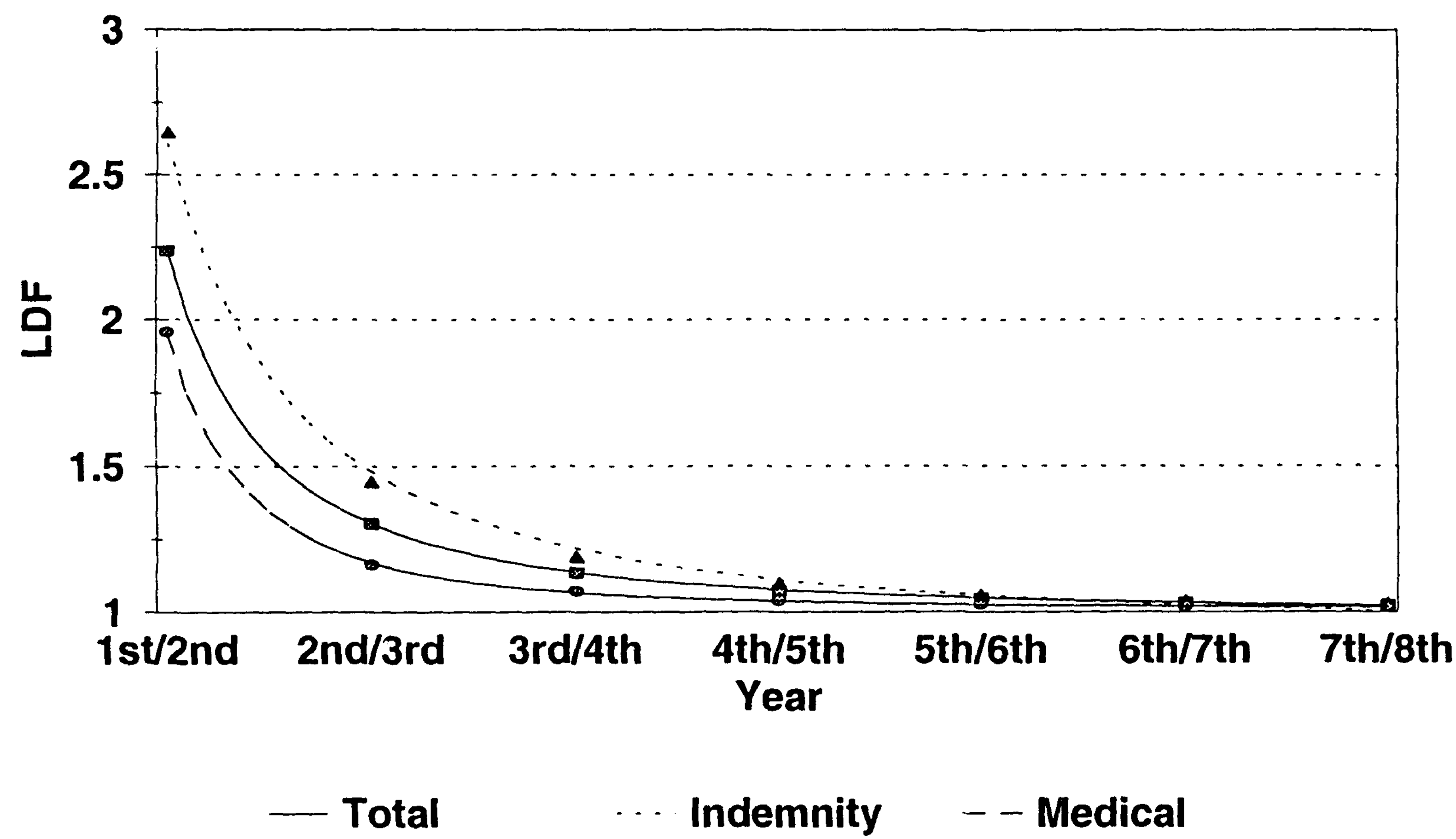
where f(i,j) is an appropriate deterministic function of the cohort year index and the development year j. For the time being, the error terms ε_{ij} are assumed to be independent Gaussian random variables with common zero mean and finite variance $\sigma^2 w_j > 0$, allowed to depend on j in such a way that

$$\sum_{j=1}^{\infty} w_j < \infty. \tag{2.2}$$

In (2.2), it is assumed that the weights $w_j > 0$ are known, but the parameter $\sigma^2 > 0$ is unknown.

The exact form of the function f in (2.1) is unknown, but approximations are easily derived based on plausible analysis and empirical evidence. For a fixed cohort i, the LDFs must approach 1 as development year j increases. This is true because eventually the claims for each claimant in the ith cohort must cease. Factors affecting this include mortality, claim settlement, and injury recovery. Empirical evidence, such as data collected by the National Council on Compensation Insurance,[2] suggests that for fixed i, exp[f(i,j)] should decrease monotonically and smoothly to 1 as $j \to \infty$. Equivalently, f(i,j) should decrease monotonically and smoothly to 0 as $j \to \infty$. To illustrate this, Figure 2.1 shows plots of LDFs versus year for accident cohorts from 1979 through 1990 as reported by the National Council on Compensation Insurance[2] from data reported by private insurers providing workers' compensation coverage in 37 states. These LDF plots are for indemnity, medical, and total costs. Each data point is computed from a five-year average.

Figure 2.1. Nationwide Loss Development Factors From the National Council on Compensation Insurance[2]



Therefore, regardless of the exact form of f, it can be postulated that for fixed i, f has an asymptotic expansion (as $j \to \infty$) of the form

$$f(i,j) \sim A_0 + A/j + B/j^2 + C/j^3 + D/j^4 + \dots \qquad (2.3)$$

(Here, the notation $g(j) \sim h(j)$ as $j \to \infty$ means that $\lim_{j\to\infty} g(j)/h(j) = 1$.) The leading two terms in (2.3) can be eliminated as follows. Ultimately, there must be an upper bound for $C_{i,j}$ as j increases. That is, cohort i eventually ceases to generate further costs. The model (2.1) entails, by iteration, that

$$C_{i,j} = C_{i,1} \prod_{k=2}^{j} \exp(Y_{ik}) = C_{i,1} \exp\left(\sum_{k=2}^{j} Y_{ik}\right) = \exp\left(\sum_{k=1}^{j} f(i,k) + \sum_{k=1}^{j} \varepsilon_{ik}\right). \qquad (2.4)$$

Because of (2.2), the random series $\sum_{k=1}^{\infty} \varepsilon_{ik}$ converges (to a random variable that is finite) with probability 1. By (2.3),

$$\sum_{k=1}^{j} f(i,j) \sim \sum_{k=1}^{j} \left[A_0 + A/k + B/k^2 + C/k^3 + D/k^4 + \dots\right]$$

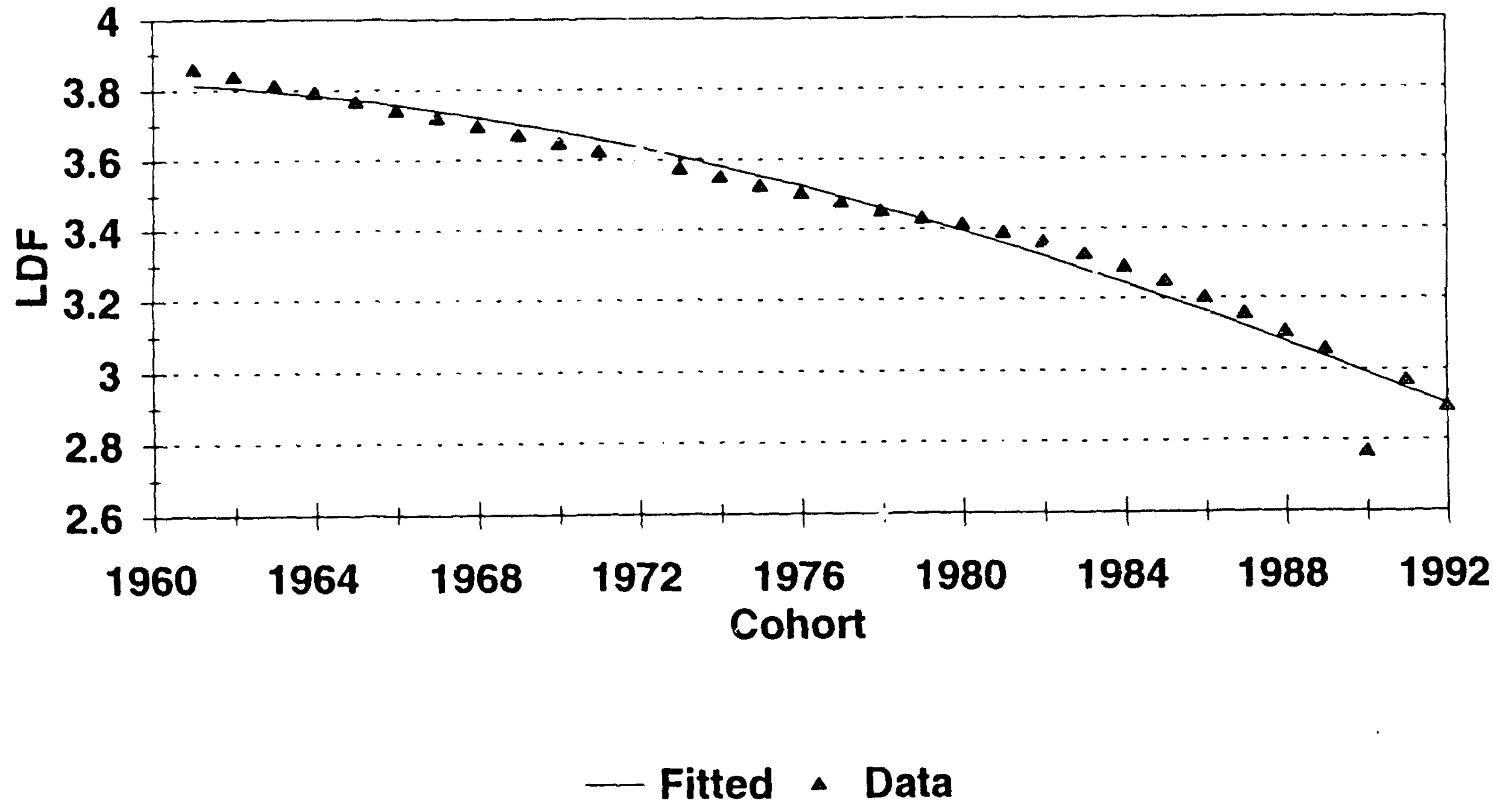
and since $A_0\, j \to \infty$ and $\sum_{k=1}^{j} 1/k \sim \log(j) \to \infty$ as $j \to \infty$ while $\sum_{k=1}^{\infty} (1/k)^m < \infty$ for $m > 1$, then $\lim_{j\to\infty} \sum_{k=1}^{j} f(i,j)$ will be finite if and only if $A_0 = A = 0$. Thus, setting $A_0 = A = 0$ in (2.3) and truncating the expansion after a finite number of terms should present a fairly accurate approximation of f(i,j) for fixed i. To verify this hypothesis, Figure 2.1 shows the model (2.3) fit to the data from the National Council on Compensation Insurance[2] with $A_0 = A = 0$ and terms beyond order 4 neglected.

From Figure 2.1 it may be concluded that qualitatively, the model (2.3) is a plausible model for LDF data. It is noted that the National Council on Compensation Insurance data[2] is based on commercially managed payment systems that are very different from the mechanisms present in the U.S. Department of the Navy. In particular, the rate of claim settlement is much

higher commercially. Therefore, it would not necessarily be true that the parameters in model (2.3) that apply to the National Council on Compensation Insurance data[2] would be approximately the same as those that apply to the U.S. Department of the Navy data. In fact, with much slower claim settlement, a better empirical fit of (2.3) to the U.S. Department of the Navy data may be achieved by including the terms A_0 and A/j, and relaxing the condition (2.2). While theoretically it should be true that $A_0 = A = 0$ and (2.2) holds, real data in which claim settlement is slow (i.e., the approach of f(i,j) to 0 as $j \to \infty$ is apparently slow) may not be extensive enough to effect a good fit to (2.3) with these conditions forced.

The study of the variation of f(i,j) for fixed j is based mostly on empirical observation. Evidence from both Miccolis[3] and the National Council on Compensation Insurance[2] suggests there is a slight decreasing trend in LDF as cohort year increases for fixed j. Figure 2.2 shows LDFs from Miccolis[3] for j=2 varying from the 1961 cohort to the 1992 cohort. A quadratic trend fit by least squares is also shown in Figure 2.2. (It is noted that the data representing the 1961 through 1989 cohorts were imputed in Miccolis,[3] as discussed in the introduction.) Of course, as j increases, the total variability of *f(i,j) diminishes, so there is the need to allow* interaction (i.e., deviation from purely additive effects of the variables i and j) between the independent variables i and j.

Figure 2.2. 1st/2nd LDF Versus Cohort From 1961 to 1962



Assuming f is a smooth function of i and 1/j, then regardless of the true form of f, a bivariate Taylor expansion of f should provide an approximation (in this case a sum of monomials in the variables i and 1/j) that is reasonably accurate. Retaining terms only up to order 4 in 1/j and order 2 in i, this leads to the consideration of a parametric model for f of the form

$$f(i,j) = \sum_{p=0}^{2} A_{p0}\, i^p + \sum_{q=1}^{4} A_{0q}\, (1/j^q) + \sum_{p=1}^{2}\sum_{q=1}^{4} A_{pq}\, i^p\, (1/j^q). \tag{2.5}$$

In fitting the model (2.5) to actual data, it will often turn out that not all terms are needed. For example, a stepwise regression procedure will systematically add only the terms necessary in (2.5) to effect a good fit to the data without overfitting or underfitting. For further discussion of over- and underfitting, see the report by Angus.[4]

2.1 Matrix Formulation of the Model

The LDF model developed thus far may be expressed as a standard linear model as follows:

First, for $j \geq 2$ and $i \geq 1$, let $v(i,j)^t$ denote the row vector

$$v(i,j)^t = \begin{pmatrix} 1 & i & i^2 & 1/j & 1/j^2 & 1/j^3 & 1/j^4 & i/j & i/j^2 & i/j^3 & i/j^4 & i^2/j & i^2/j^2 & i^2/j^3 & i^2/j^4 \end{pmatrix}$$

and let β denote the column vector

$$\beta = \begin{pmatrix} A_{00} & A_{10} & A_{20} & A_{01} & A_{02} & A_{03} & A_{04} & A_{11} & A_{12} & A_{13} & A_{14} & A_{21} & A_{22} & A_{23} & A_{24} \end{pmatrix}^t$$

with M^t signifying the transpose of the matrix M. Also, let

$$Y = \left(Y_{12}\ Y_{22} \cdots Y_{N-1,2} \,\middle|\, Y_{13}\ Y_{23} \cdots Y_{N-2,3} \,\middle|\, \cdots \,\middle|\, Y_{1,N-1}\ Y_{2,N-1} \,\middle|\, Y_{1,N} \right)^t$$

$$\varepsilon = \left(\varepsilon_{12}\ \varepsilon_{22} \cdots \varepsilon_{N-1,2} \,\middle|\, \cdots \,\middle|\, \varepsilon_{1,N-1}\ \varepsilon_{2,N-1} \,\middle|\, \varepsilon_{1,N}\right)^t,$$

and X be the n by 15 matrix whose transpose is given by

$$X^t = \left(v(1,2)\ v(2,2) \ldots v(N-1,2) \,\middle|\, v(1,3)\ v(2,3) \ldots v(N-2,3) \,\middle|\, \ldots \,\middle|\, v(1,N-1)\ v(2,N-1) \,\middle|\, v(1,N)\right),$$

where

$$n = N(N-1)/2$$

is the total number of components of Y. Finally, let W be the n by n diagonal matrix

$$W = \operatorname{diag}\left(w_2\ w_2 \ldots w_2 \middle| w_3\ w_3 \ldots w_3 \middle| \ldots \middle| w_{N-1}\ w_{N-1} \middle| w_N\right),$$

where the first block containing w_2 is of length N-1, the second block containing w_3 is of length N-2, and so on. Then using (2.5) the model (2.1) may be written as

$$Y_{ij} = v(i,j)^t\beta + \varepsilon_{ij} \tag{2.6}$$

or in matrix notation as

$$Y = X\beta + \varepsilon. \tag{2.7}$$

The error vector in (2.7) is transformed to one with homogeneous variances by premultiplying by $W^{-1/2}$ to yield

$$W^{-1/2}Y = W^{-1/2}X\beta + \varepsilon^* \tag{2.8}$$

where now ε^* is n-variate Gaussian with zero mean and variance matrix $\sigma^2 I$ (I being the n by n

identity matrix). Standard linear statistical model theory, such as that found in the text by Arnold,[5] now provides the result that the maximum likelihood and best linear unbiased estimator of β is the weighted least squares estimator given by

$$\hat{\beta} = (X^t W^{-1} X)^{-1} X^t W^{-1} Y \tag{2.9}$$

and an unbiased estimator of σ^2 is

$$\begin{aligned} \hat{\sigma}^2 &= \frac{1}{n-p} Y^t \left(W^{-1} - W^{-1} X (X^t W^{-1} X)^{-1} X^t W^{-1} \right) Y \\ &= \frac{1}{n-p} \left(Y - X\hat{\beta} \right)^t W^{-1} \left(Y - X\hat{\beta} \right) \end{aligned} \tag{2.10}$$

where p is the number of unknown parameters in β. (It is possible that not all 15 of the terms in (2.5) will be needed. For the terms not needed, the corresponding columns of X and parameters in β are removed, and the dimensions of X and β adjusted accordingly. It is assumed that these adjustments have been made throughout the analysis.)

Additional facts from linear statistical model theory that will be useful in developing prediction intervals are that

$$\hat{\beta} \overset{d}{=} N\left(\beta, \sigma^2 (X^t W^{-1} X)^{-1}\right), \tag{2.11}$$

$$(n-p)\hat{\sigma}^2/\sigma^2 \overset{d}{=} \chi^2(n-p), \tag{2.12}$$

and that $\hat{\beta}$ and $\hat{\sigma}^2$ are statistically independent. Here, $N(\mu, \Sigma)$ signifies a random variable that is normally distributed with mean (scalar or vector) μ and variance (scalar or matrix) Σ, and $\chi^2(m)$ signifies a random variable with a chi-squared distribution with m degrees of freedom. The notation $X \overset{d}{=} Y$ means that the random variables X and Y have the same distribution.

2.2 Point Prediction

The first step in prediction of costs is the prediction of the missing log-LDFs. Define

the predicted value of Y_{ij} by

$$\hat{Y}_{ij} = \begin{cases} Y_{ij} & \text{if } Y_{ij} \text{ is observed;} \\ v(i,j)^t \hat{\beta} & \text{otherwise } (j \geq 2), \end{cases} \tag{2.13a}$$

so that the predicted LDF going form year j-1 to year j for cohort i is

$$\hat{LDF}_{ij} = \exp(\hat{Y}_{ij}) \tag{2.13b}$$

and the predicted cumulative LDF (CLDF) going from year 1 to year j is

$$\hat{CLDF}_{ij} = \prod_{k=2}^{j} \hat{LDF}_{ik} = \exp\left(\sum_{k=2}^{j} \hat{Y}_{ik}\right). \tag{2.13c}$$

(This usage of the term "cumulative LDF" is slightly different from common usage in actuarial literature. Typically, as defined by the National Council on Compensation Insurance,[2] cumulative LDF is computed from a given base year to an "ultimate." This "ultimate" corresponds to the limit as $j \to \infty$ in this model. This approach was not adopted here, since no "ultimate" costs were available to aid in fitting the models, and predictions out to 32 years, as well as intermediate years, were of interest.)

In (2.13a), $\hat{\beta}$ is the estimator given in (2.9). Recalling that the model (2.6) or (2.7) entails that

$$C_{i,j} = C_{i,1} \exp\left(\sum_{k=2}^{j} Y_{i,k}\right) \tag{2.14}$$

for $2 \leq j \leq N-i+1$, and treating the initial costs $C_{i,1}$ as constants ($1 \leq i \leq N$), a natural predictor for missing $C_{i,j}$ is given by

$$\hat{C}_{i,j} = C_{i,1} \exp\left(\sum_{k=2}^{j} \hat{Y}_{i,k}\right) \tag{2.15}$$

with $\hat{Y}_{i,k}$ given by (2.13a). The predictions (2.15) are nearly the same as those that would be computed using the actuarial method described in the introduction except that the unknown (log) LDFs are predicted using the regression equation *structure* (2.6) *rather than simple* averaging over (nonhomogeneous) cohorts. Thus, (2.15) should yield more accurate predictions.

To illustrate the development of the predictions in (2.15), the following assum[illegible] N = 4. Here, the upper left triangular (including the diagonal elements) numbers ar[illegible]ually observed and do not need to be predicted, but nevertheless, (2.15) reduces to the actual observed values. The lower right part of the matrix illustrates exactly which (log) LDFs are predicted using the estimated regression equation.

$$
\begin{array}{llll}
C_{1,1} & C_{1,1}\exp(Y_{12}) & C_{1,1}\exp(Y_{12}+Y_{13}) & C_{1,1}\exp(Y_{12}+Y_{13}+Y_{14}) \\
C_{2,1} & C_{2,1}\exp(Y_{22}) & C_{2,1}\exp(Y_{22}+Y_{23}) & C_{2,1}\exp(Y_{22}+Y_{23}+\hat{Y}_{24}) \\
C_{3,1} & C_{3,1}\exp(Y_{32}) & C_{3,1}\exp(Y_{32}+\hat{Y}_{33}) & C_{3,1}\exp(Y_{32}+\hat{Y}_{33}+\hat{Y}_{34}) \\
C_{4,1} & C_{4,1}\exp(\hat{Y}_{42}) & C_{4,1}\exp(\hat{Y}_{42}+\hat{Y}_{43}) & C_{4,1}\exp(\hat{Y}_{42}+\hat{Y}_{43}+\hat{Y}_{44})
\end{array}
$$

It is apparent from (2.13) through (2.15) that the predicted costs also satisfy the recursion

$$\hat{C}_{i,j} = \hat{C}_{i,j-1}\exp(\hat{Y}_{ij}) \tag{2.16}$$

where $j \geq 2$, which is exactly the "loss development principle;" the predicted cumulative cost for year j is the predicted cumulative cost for year j-1 times the estimated LDF for year j-1 to j.

2.3 Interval Prediction

Suppose that in addition to predicting $C_{i,j}$, it is necessary to find random variables L and U so that for a fixed, preselected probability $1-\alpha$, $P\{L < C_{i,j} < U\} = 1-\alpha$. Here, it is assumed that $i \leq N$ and $j > N-i+1$. (If $j \leq N-i+1$ then $C_{i,j}$ is observed and there is nothing to predict.) The quantity given by

$$\left(\frac{C_{i,j}}{\hat{C}_{i,j}}\right)^{1/\hat{\sigma}} = \exp\left(\sum_{k=N-i+2}^{j}(Y_{ik}-\hat{Y}_{ik})/\hat{\sigma}\right) \tag{2.17}$$

is pivotal; that is, it has a distribution that does not depend on any unknown parameters. In (2.17), $\hat{\sigma}^2$ is given by (2.10) and $\hat{Y}_{ik}$ is given by (2.13a).

To see why (2.17) is pivotal, notice that Y_{ik}, $k \geq N-i+2$, are not observed in the current data and are considered future observations. Thus, by the assumptions underlying (2.6), it follows that Y_{ik}, $k \geq N-i+2$, are independent of $\hat{Y}_{ik}$, since the latter are functions of the current data. Since

$$Y_{ik}-\hat{Y}_{ik} = Y_{ik}-v(i,k)^t\,\hat{\beta} = v(i,k)^t(\beta-\hat{\beta})+\varepsilon_{ik}$$

where $\varepsilon_{ik} \overset{d}{=} N(0, \sigma^2 w_k)$ is independent of $\hat{\beta}$, it follows that

$$\sum_{k=N-i+2}^{j}(Y_{ik}-\hat{Y}_{ik}) = \left(\sum_{k=N-i+2}^{j}v(i,k)^t\right)(\beta-\hat{\beta}) + \sum_{k=N-i+2}^{j}\varepsilon_{ik}$$

and hence from (2.11)

$$\sum_{k=N-i+2}^{j}(Y_{ik}-\hat{Y}_{ik}) \overset{d}{=} N\left(0, \sigma^2\tau_{ij}^2\right) \tag{2.18}$$

where

$$\tau_{ij}^2 = \sum_{k=N-i+2}^{j}w_k + \left(\sum_{k=N-i+2}^{j}v(i,k)^t\right)(X^tW^{-1}X)^{-1}\left(\sum_{k=N-i+2}^{j}v(i,k)^t\right)^t. \tag{2.19}$$

It follows from independence and (2.12) that

$$(1/\tau_{ij})\sum_{k=N-i+2}^{j}(Y_{ik}-\hat{Y}_{ik})/\hat{\sigma} \overset{d}{=} t(n\text{-}p), \tag{2.20}$$

that is, has a t-distribution with n-p degrees of freedom. Denoting by $t_{\upsilon}(m)$ the υ quantile of the t-distribution with m degrees of freedom, (2.20) implies that

$$P\left\{\exp\left(\tau_{ij}t_{\alpha/2}(n\text{-}p)\right) < \left(C_{i,j}/\hat{C}_{i,j}\right)^{1/\hat{\sigma}} < \exp\left(\tau_{ij}t_{1-\alpha/2}(n\text{-}p)\right)\right\} = 1\text{-}\alpha, \tag{2.21}$$

which in turn implies that

$$P\left\{\hat{C}_{i,j}\exp\left(\tau_{ij}\hat{\sigma}\,t_{\alpha/2}(n\text{-}p)\right) < C_{i,j} < \hat{C}_{i,j}\exp\left(\tau_{ij}\hat{\sigma}\,t_{1-\alpha/2}(n\text{-}p)\right)\right\} = 1\text{-}\alpha. \tag{2.22}$$

Relation (2.22) says that with probability 1-α, the future cost $C_{i,j}$ will lie in the interval

$$\left(\hat{C}_{i,j}\exp\left(\tau_{ij}\hat{\sigma}\,t_{\alpha/2}(n\text{-}p)\right), \hat{C}_{i,j}\exp\left(\tau_{ij}\hat{\sigma}\,t_{1-\alpha/2}(n\text{-}p)\right)\right). \tag{2.23a}$$

In (2.23a), it is tacitly assumed that $0 < \alpha < 1$. Notice that the left endpoint is the point prediction $\hat{C}_{i,j}$ multiplied by a factor less than 1 (i.e. reduced, since $t_{\upsilon}(m) < 0$ for $\upsilon < 1/2$) and the right endpoint is the point prediction $\hat{C}_{i,j}$ multiplied by a factor greater than 1 (i.e. increased, since $t_{\upsilon}(m) > 0$ for $\upsilon > 1/2$).

By taking $C_{i,1} \equiv 1$ for all i, the analysis that led to expression (2.33a) also provides (1-α) prediction intervals for cumulative LDF via

$$\left(\exp\left(\sum_{k=2}^{j}\hat{Y}_{ik} + \tau_{ij}\hat{\sigma}\,t_{\alpha/2}(n\text{-}p)\right), \exp\left(\sum_{k=2}^{j}\hat{Y}_{ik} + \tau_{ij}\hat{\sigma}\,t_{1-\alpha/2}(n\text{-}p)\right)\right). \tag{2.23b}$$

Similarly, prediction limits for individual LDFs are easily developed. Letting

$$r_{ij}^2 = w_j + v(i,j)^t (X^t W^{-1} X)^{-1} v(i,j) \tag{2.23c}$$

it follows that a 1-α prediction interval for the LDF $\exp(Y_{ij})$ is (assuming $j > N-i+1$)

$$\left(\exp\left(\hat{Y}_{ij} + r_{ij}\hat{\sigma}\, t_{\alpha/2}(n-p)\right), \exp\left(\hat{Y}_{ij} + r_{ij}\hat{\sigma}\, t_{1-\alpha/2}(n-p)\right)\right). \tag{2.23d}$$

2.4 Geometric Average Model

In this section it is assumed that the function f(i,j) in (2.1) does not depend on i (i.e, that LDFs are homogeneous across cohorts but may change with development year j). Thus, assume that for all i,

$$f(i,j) = \mu(j). \tag{2.24}$$

This would partly justify the actuarial method of prediction discussed in the introduction, but instead of taking simple averages of like-LDFs as advocated in the actuarial method, the following will make a case for using geometric averages instead.

Instead of parametric modeling of the variation in expected log-LDF over j as in the previous sections, it is also possible to treat each value of $\mu(\bullet)$ in (2.24) as a parameter to be estimated. The disadvantage here is that predictions will not be available for $C_{i,j}$, for $j > N$, since observed LDFs will not be available in this range. It could be argued, however, that prediction beyond the development year range of the data is not advisable anyway.

Another difficulty with the approach of this section is that there is loss in efficiency by introducing so many more parameters. In the previous method, no more than 15 parameters (not including σ^2) are estimated from the data while in the present model, there must be N-1 parameters estimated (namely $\mu(2)$, ..., $\mu(N)$). Loosely speaking, for $N > 16$ and to achieve a given level of precision, more data will be needed for the model of this section than for the regression model of section 2.2.

Under this set up, the parameters $\mu(j)$ are estimated by

$$\hat{\mu}(j) = \frac{1}{N-j+1}\sum_{r=1}^{N-j+1} Y_{rj}. \tag{2.25}$$

Predictions $\hat{C}^*_{i,j}$ of unknown costs $C_{i,j}$, $2 \le j \le N$, are constructed as before using

$$\hat{C}^*_{i,j} = C_{i,1} \exp\left(\sum_{k=2}^{j} \hat{Y}^*_{i,k}\right) \tag{2.26}$$

except the predicted Y_{ij}s are now computed via

$$\hat{Y}^*_{ij} = \begin{cases} Y_{ij} & \text{if } Y_{ij} \text{ is observed;} \\ \hat{\mu}(j) & \text{otherwise } (2 \le j \le N). \end{cases} \tag{2.27a}$$

As before, a prediction of LDF from year j-1 to year j for cohort i becomes

$$\hat{L}DF^*_{ij} = \exp(\hat{Y}^*_{ij}) \tag{2.27b}$$

and the predicted cumulative LDF (CLDF) going from year 1 to year j is

$$\hat{C}LDF^*_{ij} = \prod_{k=2}^{j} \hat{L}DF^*_{ik} = \exp\left(\sum_{k=2}^{j} \hat{Y}^*_{ik}\right). \tag{2.27c}$$

The parameter σ^2 is now unbiasedly estimated by

$$\tilde{\sigma}^2 = \frac{2}{(N-1)(N-2)} \sum_{j=2}^{N-1} \frac{1}{w_j} \sum_{i=1}^{N-j+1} \left(Y_{ij} - \hat{\mu}(j)\right)^2. \tag{2.28}$$

The quantity

$$\left(\frac{C_{i,j}}{\hat{C}^*_{i,j}}\right)^{1/\tilde{\sigma}} = \exp\left(\sum_{k=N-i+2}^{j} (Y_{ik} - \hat{Y}^*_{ik})/\tilde{\sigma}\right) \tag{2.29}$$

is pivotal. To see this, standard Gaussian linear model theory applies allowing one to conclude that

$$\sum_{k=N-i+2}^{j} (Y_{ik} - \hat{Y}^*_{ik}) \stackrel{d}{=} N\left(0, \sigma^2 \rho^2_{ij}\right) \tag{2.30}$$

where

$$\rho^2_{ij} = \sum_{k=N-i+2}^{j} w_k + \sum_{k=N-i+2}^{j} \frac{w_k}{N-k+1}. \tag{2.31}$$

and that

$$\frac{(N-1)(N-2)\tilde{\sigma}^2}{2\sigma^2} \stackrel{d}{=} \chi^2((N-1)(N-2)/2) \tag{2.32}$$

independently of (2.30), so that

$$(1/\rho_{ij}) \sum_{k=N-i+2}^{j} (Y_{ik} - \hat{Y}^*_{ik})/\tilde{\sigma} \stackrel{d}{=} t(n-(N-1)) \tag{2.33}$$

where $n = N(N-1)/2$ as before.

At this point, it is noteworthy to compare (2.33) with (2.20). In (2.20) the degrees of freedom are n-p with $p \leq 15$, regardless of the size of N, while in (2.33) the degrees of freedom are n-(N-1), which eventually (as N grows) is much smaller than n-p. Thus, (2.33) will be more variable than (2.20) and lead to wider prediction intervals than (2.20). On the other hand, for $N < 6$, n-p can be less than or equal to zero and then the parametric method based on (2.5) would not be applicable (that is, there are too many parameters and not enough data). In the present

model, n-(N-1) = (N-1)(N-2)/2 > 0 as long as N > 2, so it would be applicable for smaller N provided the assumption (2.24) can be made and predictions beyond j = N are not needed.

Continuing with the development of a prediction interval, (2.33) leads to

$$P\left\{\exp\left(\rho_{ij}t_{\alpha/2}(n-(N-1))\right) < \left(C_{i,j}/\hat{C}^*_{i,j}\right)^{1/\tilde{\sigma}} < \exp\left(\rho_{ij}t_{1-\alpha/2}(n-(N-1))\right)\right\} = 1-\alpha, \tag{2.34}$$

which in turn implies that

$$P\left\{\hat{C}^*_{i,j}\exp\left(\rho_{ij}\tilde{\sigma}\,t_{\alpha/2}(n-(N-1))\right) < C_{i,j} < \hat{C}^*_{i,j}\exp\left(\rho_{ij}\tilde{\sigma}\,t_{1-\alpha/2}(n-(N-1))\right)\right\} = 1-\alpha. \tag{2.35}$$

Relation (2.35) says that with probability $1-\alpha$, the future cost $C_{i,j}$ will lie in the interval

$$\left(\hat{C}^*_{i,j}\exp\left(\rho_{ij}\tilde{\sigma}\,t_{\alpha/2}(n-(N-1))\right), \hat{C}^*_{i,j}\exp\left(\rho_{ij}\tilde{\sigma}\,t_{1-\alpha/2}(n-(N-1))\right)\right). \tag{2.36a}$$

In (2.36a), it is tacitly assumed that $0 < \alpha < 1$. Notice that the left endpoint is the point prediction $\hat{C}^*_{i,j}$ multiplied by a factor less than 1 (i.e. reduced, since $t_\upsilon(m) < 0$ for $\upsilon < 1/2$) and the right endpoint is the point prediction $\hat{C}^*_{i,j}$ multiplied by a factor greater than 1 (i.e. increased, since $t_\upsilon(m) > 0$ for $\upsilon > 1/2$).

By taking $C_{i,1} \equiv 1$ for all i, the analysis that led to expression (2.36a) also provides $(1-\alpha)$ prediction intervals for cumulative LDF via

$$\left(\exp\left(\textstyle\sum_{k=2}^{j}\hat{Y}^*_{ik} + \rho_{ij}\tilde{\sigma}\,t_{\alpha/2}(n-(N-1))\right), \exp\left(\textstyle\sum_{k=2}^{j}\hat{Y}^*_{ik} + \rho_{ij}\tilde{\sigma}t_{1-\alpha/2}(n-(N-1))\right)\right). \tag{2.36b}$$

Similarly, prediction limits for individual LDFs are easily developed. Letting

$$s_j^2 = w_j + w_j/(N-j+1) \tag{2.36c}$$

it follows that a 1-α prediction interval for the LDF $\exp(Y_{ij})$ is (assuming $j > N-i+1$)

$$\left(\exp\left(\hat{Y}^*_{ij} + s_j\tilde{\sigma}\, t_{\alpha/2}(n-(N-1))\right), \exp\left(\hat{Y}^*_{ij} + s_j\tilde{\sigma}\, t_{1-\alpha/2}(n-(N-1))\right)\right). \tag{2.36d}$$

As before, the prediction formula can be expressed as

$$\hat{C}^*_{i,j} = \hat{C}^*_{i,j-1}\exp(\hat{Y}^*_{ij}). \tag{2.37}$$

When $N-i+1 < j \leq N$, (2.25) and (2.27a) imply that

$$\hat{C}^*_{i,j} = \hat{C}^*_{i,j-1}\left(\prod_{r=1}^{N-j+1} LDF_{rj}\right)^{1/(N-j+1)} \tag{2.38}$$

where

$$LDF_{rj} = \exp(Y_{rj}) \tag{2.39}$$

is the observed loss development factor from year j-1 to j for cohort r. Thus, this method of prediction is nearly the same as the basic actuarial method, except that simple averaging of LDFs has been replaced by geometric averaging of the LDFs, as seen in (2.38).

3. Application to U.S. Department of the Navy Workers' Compensation Claims

3.1 Application to Complete 1990 Through 1993 Cohort Data

The data in Table 1.1 of the introduction represent actual complete cohort data for 1990 through 1993 cohorts having workers' compensation claims against the U.S. Department of the Navy. This data set is small, and the LDF trend, if any, across cohorts is weak, as seen in Table 1.2. The full parametric model of sections 2.2 through 2.3 is not applicable (too many unknown parameters in that model), but the model of section 2.4 may be applied to compute predictions and prediction intervals. Using $N = 4$, $w_1 = ... = w_6 = 1$, formulae (2.28) and (2.31) yield $\tilde{\sigma} = 0.0299184$; $\rho_{24} = 2$, $\rho_{33} = 3/2$, $\rho_{34} = 7/2$, $\rho_{42} = 4/3$, $\rho_{43} = 17/6$, and $\rho_{44} = 29/6$. Here, $(N-1)(N-2)/2 = 3$. Taking $\alpha = .20$ for an 80% prediction interval, $t_{.90}(3) = 1.6377 = -t_{.10}(3)$, and (2.36) yields the prediction intervals in Table 3.1.

Table 3.1. 80% Prediction Intervals for 1991 Through 1993 Cohorts Based on Data from Table 1.1 (In Thousands of Dollars)

Cohort Year	Development Year 2			3			4		
	Lower	Pred.	Upper	Lower	Pred.	Upper	Lower	Pred.	Upper
1991							71,343	76,462	81,948
1992				59,375	63,047	66,946	73,140	80,161	87,857
1993	37,263	39,432	41,727	54,646	59,344	64,446	67,749	75,454	84,036

3.2 Application to Actuarial Data From Miccolis[3]

The somewhat wide prediction intervals in Table 3.1 reflect the fact that they are based on a very small sample. When more data are available, the regression model of sections 2.1 through 2.3 can lead to more precise predictions. To illustrate their use, these models were applied to data found in exhibit 8, sheets 3a and 3b of the report by Miccolis.[3] These data represent (mostly imputed) total cumulative costs (indemnity plus medical) for cohorts from

1961 through 1993, and development years 1 to 32. Because complete data were unavailable for cohort years prior to 1990, Miccolis[3] employed a variety of actuarial techniques to estimate claim counts and average costs per claim for the cohorts prior to 1990 using all available data. Thus, the reader shou'd keep in mind that data employed in this example contain a great deal of imputed values and are smoother (i.e. exhibit less fluctuation) than a completely real data set. Consequently, the variance estimate (2.10) will be smaller than expected, leading to prediction intervals that are narrower than could be expected from comparable real data.

For this application, there are N = 33 cohorts, and the cumulative cost data are summarized in Table 3.2. The data employed by the models of sections 2.1 through 2.3 are the log-LDFs, namely $Y_{ij} = \log(C_{i,j}/C_{i,j-1})$, $1 \le i \le 34\text{-}j$, $2 \le j \le 32$, $C_{i,j}$ being the cumulative cost of cohort i through development year j. Cohort years 1961 through 1993 correspond to i = 1 through i = 33, respectively.

Table 3.2. Cumulative Cost Data From Miccolis[3]

Cohort	Development Year 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1961	928	3583	6214	8611	10829	13018	15067	17106	19047	20903	22549	24106	25723	27256	28822	30326
1962	1016	3901	6748	9342	11741	14109	16324	18528	20626	22632	24410	26093	27840	29498	31188	32813
1963	978	3729	6436	8900	11178	13426	15528	17620	19611	21515	23202	24799	26456	28028	29632	31173
1964	1190	4513	7770	10733	13471	16172	18698	21210	23602	25889	27914	29832	31822	33710	35635	37485
1965	1380	5201	8932	12324	15457	18547	21437	24310	27045	29659	31975	34168	36443	38601	40801	42916
1966	1504	5631	9646	13293	16661	19981	23085	26172	29109	31917	34404	36758	39202	41519	43881	46151
1967	1830	6810	11635	16015	20058	24043	27768	31471	34994	38361	41344	44167	47098	49875	52707	55428
1968	2409	8907	15177	20864	26113	31284	36115	40919	45487	49855	53721	57383	61182	64784	68455	71983
1969	2666	9793	16640	22846	28572	34211	39478	44714	49693	54453	58665	62656	66797	70724	74725	78570
1970	3145	11479	19451	26671	33329	39884	46006	52090	57874	63403	68296	72934	77747	82312	86961	91430
1971	3442	12479	21084	28872	36050	43115	49711	56266	62497	68451	73722	79721	83910	88833	93844	98661
1972	3998	14398	24256	33170	41385	49463	57006	64499	71620	78429	84457	90178	96118	101754	107489	113001
1973	4528	16198	27208	37155	46314	55324	63730	72081	80020	87612	94335	100722	107355	113650	120050	126202
1974	4997	17753	29730	40541	50492	60276	69403	78472	87096	95348	102655	109605	116824	123677	130638	137332
1975	4933	17406	29059	39569	49237	58739	67636	76421	84805	92831	99939	106709	113742	120421	127198	134367
1976	6015	21079	35081	47697	59297	70703	81348	91935	102009	111655	120201	128352	136824	144870	153328	161652
1977	6762	23534	39042	53001	65840	78469	90259	101989	113153	123850	133329	142386	151804	159942	168548	176531
1978	5054	17465	28981	39157	48611	57913	66599	75244	83476	91369	98365	105063	112561	120199	127231	134063
1979	5496	18860	31101	42118	52256	62233	71552	80833	89673	98155	105677	112901	120125	127578	134432	
1980	5032	17179	28245	38200	47363	56383	64810	73207	81211	88897	95324	101554	107480	113671		
1981	5039	17090	27992	37794	46816	55701	64004	72284	80181	87482	93968	100514	107375			
1982	4947	16637	27123	36544	45217	53761	61748	69720	76804	83295	89751	96564				
1983	5205	17333	28102	37769	46669	55440	63643	72835	80626	88761	96594					
1984	5771	19000	30610	41019	50605	60056	69330	78291	87468	96620						
1985	6308	20505	32796	43803	53941	64819	74592	83865	93178							
1986	7840	25133	39876	53058	65749	77771	88830	99081								
1987	8070	25485	40075	54074	66393	77710	88070									
1988	10059	31262	49627	64956	79078	92009										
1989	12199	37324	55962	72550	86373											
1990	14955	41424	62897	79971												
1991	13566	40314	60137													
1992	14468	41892														
1993	13702															

Table 3.2, continued. Cumulative Cost Data From Miccolis[3]

	Development Year															
Cohort	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
1961	31749	33300	34826	36270	37683	39073	40390	41647	42821	44001	45164	46267	47309	48377	49385	50387
1962	34351	36028	37678	39239	40768	42271	43696	45062	46338	47620	48887	50089	51169	52245	53291	54297
1963	32633	34224	35790	37272	38723	40152	41504	42807	44027	45255	46466	47765	48862	49967	51138	
1964	39238	41150	43031	44811	46556	48273	49906	51482	52960	54447	55941	57481	58823	60167		
1965	44919	47106	49258	51295	53290	55256	57133	58949	60652	62418	64207	65828	67539			
1966	48301	50650	52962	55151	57295	59418	61450	63420	65333	67048	68721	70250				
1967	58007	60825	63599	66225	68811	71372	73828	76147	78189	80227	82376					
1968	75328	78985	82584	85995	89368	92718	96119	99181	102332	105501						
1969	82216	86207	90135	93875	97585	101029	104153	107314	110385							
1970	95670	100314	104907	109284	113335	117520	121609	125594								
1971	103256	108294	113278	117629	122206	126387	130630									
1972	118287	124091	129770	135281	140957	146655										
1973	132135	138859	145385	152045	158245											
1974	144054	151278	158040	164770												
1975	140984	147680	154350													
1976	169497	177538														
1977	184125															

Using stepwise regression, the model (2.6) was fit without weighting (i.e. using $w_k=1$ for all k). Only seven terms in (2.5) were needed, and the best model therefore had p = 7 and

$$Y_{ij} = v(i,j)^t\beta + \varepsilon_{ij} \tag{3.1}$$

with

$$v(i,j)^t = \left(1 \;\; 1/j^2 \;\; 1/j^3 \;\; 1/j^4 \;\; i/j^4 \;\; i^2/j^2 \;\; i^2/j^3\right) \tag{3.2}$$

and

$$\beta^t = \left(A_{00} \;\; A_{02} \;\; A_{03} \;\; A_{04} \;\; A_{14} \;\; A_{22} \;\; A_{23}\right). \tag{3.3}$$

The estimates of β and σ^2 were

$$\hat{\beta}^t = \left(.0214289 \quad 8.6704 \quad -22.5927 \quad 31.5725 \quad -.016130 \quad -.00123026 \quad .0005548\right)$$

$$\hat{\sigma}^2 = (.007093)^2 \tag{3.4}$$

and the fit yielded an R^2 value of 99.9%.

Miccolis[3] provided predictions of the costs $C_{i,j}$ for $35-i \le j \le 32$, $3 \le i \le 33$. Converting those and the data $C_{i,j}$, $1 \le i \le 34\text{-}j$, $1 \le j \le 32$, by the formula $LDF_{ij} = C_{i,j}/C_{i,j-1}$, and plotting LDF_{ij} versus (i, j) yields Figure 3.1. Using the fitted model (3.1) through (3.4), and using the methods in sections 2.2 to compute predictions $\hat{C}_{i,j}$, the surface in Figure 3.2 was produced. It is readily seen that not only does the model (3.1)-(3.4) provide an excellent fit to the data, but it also produces predictions of the LDFs (outside of the original data) that are close to the predictions produced by Miccolis[3] using different actuarial methods. It is emphasized here that

the predicted LDFs from Miccolis[3] (lower right part of the data matrix of Table 3.2) were NOT part of the data used to fit the regression model (3.1)-(3.4).

Figure 3.1. Loss Development Factors (Observed and Predicted From Miccolis[3])

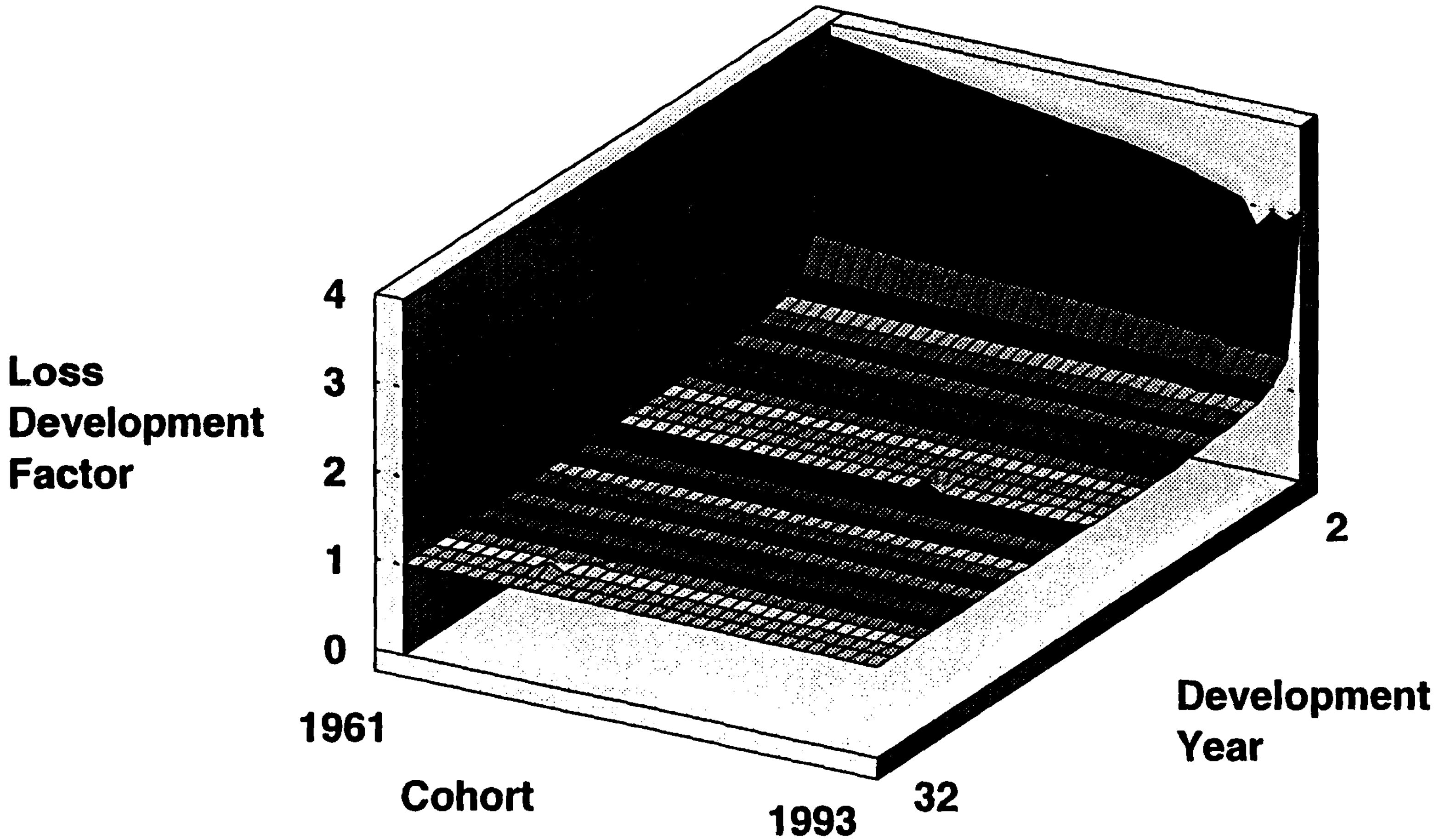
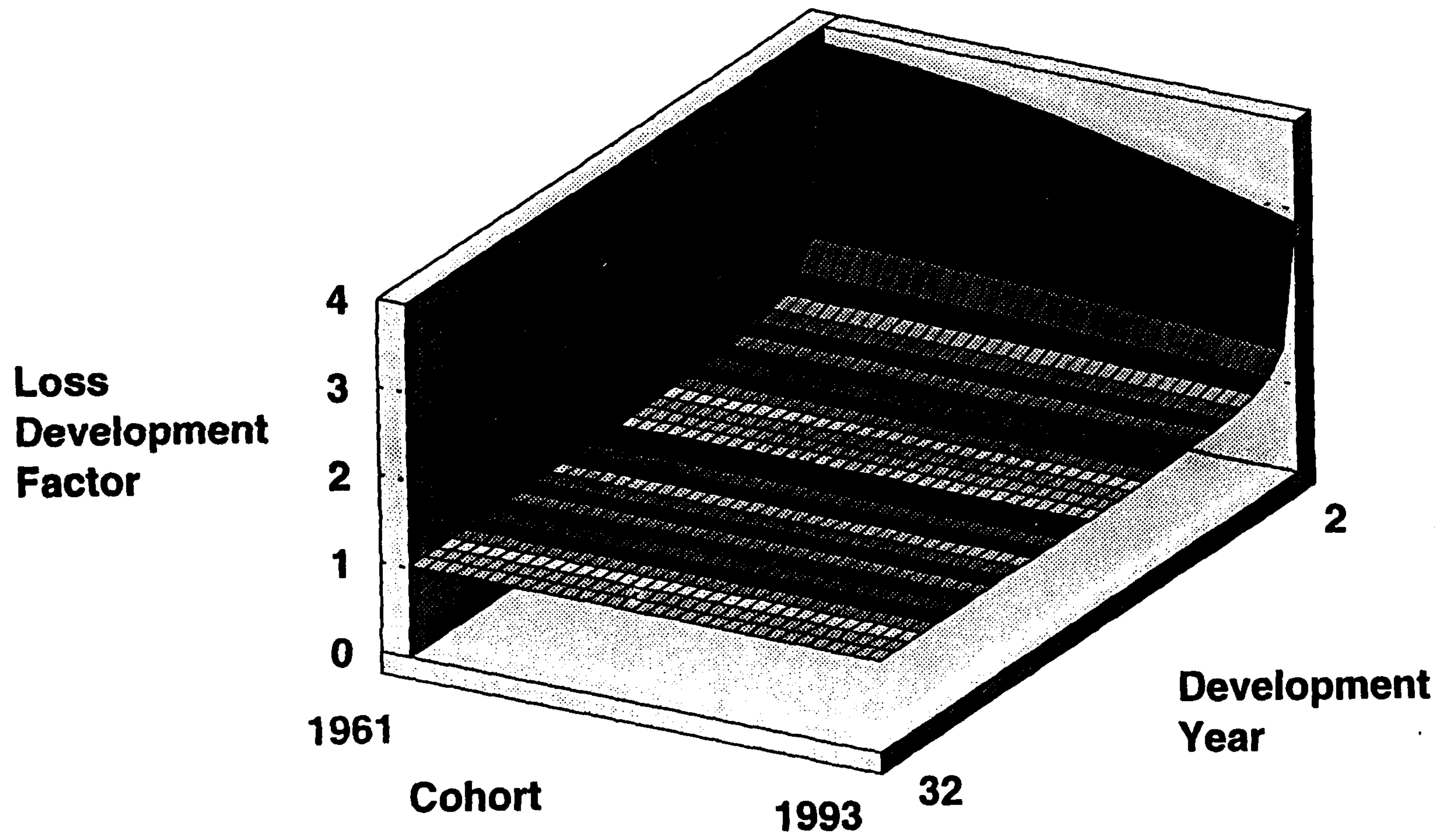


Figure 3.2. Loss Development Factors
(Using Methods of Sections 2.2 & 2.3)



Using (2.23b) and (2.36b), predictions and prediction interval endpoints were computed for cumulative LDF (CLDF) and LDF. (Recall that an LDF is always for consecutive years in this report, while cumulative LDF describes the loss development from the first year to the year specified.) These are shown in Tables 3.3 and 3.4. Table 3.5 shows all the predicted costs up to year 32 for the 1989 through 1993 cohorts, along with lower and upper endpoints for prediction intervals. Figure 3.3 shows the cumulative cost predictions for the 1990 cohort, along with upper and lower endpoints of a 95% prediction interval. The costs prior to year 5 are known for the 1990 cohort, so there are no prediction intervals prior to year 5.

Recall that the LDF and CLDF are known exactly up to and including years 5, 4, 3, and 2 for the cohorts of years 1989, 1990, 1991, and 1992, respectively. Thus, these are not tabled. *Also, the cumulative LDF at year 5, 4, 3, and 2 has been divided out of the formulae (2.23b) and (2.36b) for cohorts 1989, 1990, 1991, and 1992, respectively.* That is, the CLDFs in Table 3.3 start with the initial year shown as the first year in the cumulative calculation. Thus, to use the tables to compute, for example, the predicted cost at development year 32 for the 1989 cohort, look up the CLDF for year 32 in Table 3.3 and multiply it by the last known cumulative cost, namely, that at year 5. This value is (in thousands) \$86,373 (from Table 3.2). Thus, the prediction is (4.2077) (\$86,373) = \$363,432 which is in agreement (within round-off error) with Table 3.5. This usage of the term cumulative LDF is slightly different from common usage in actuarial literature. See section 2.2 for a discussion of these differences.

As another example, to compute a prediction and prediction interval for the cumulative cost for the 1993 cohort at year 32, look up the predicted CLDF from Table 3.3, Cohort Year = 1993, and Year = 32. The predicted value is 26.8176. For a 95% prediction interval (with equal tail probabilities), find the .025 and .975 endpoints for year 32. These are 24.6378 and 29.1903, respectively. Finally, multiply these (the endpoints and the prediction) by the initial (year 1) cost for the 1993 cohort (from Table 3.2, \$13,702 in thousands). For this example, the predicted cost at year 32 is then \$367,455, and the 95% prediction interval is (\$337,587, \$399,965), all in thousands.

Figure 3.3. 95% Prediction Intervals
1990 Cohort (Costs in $1K)

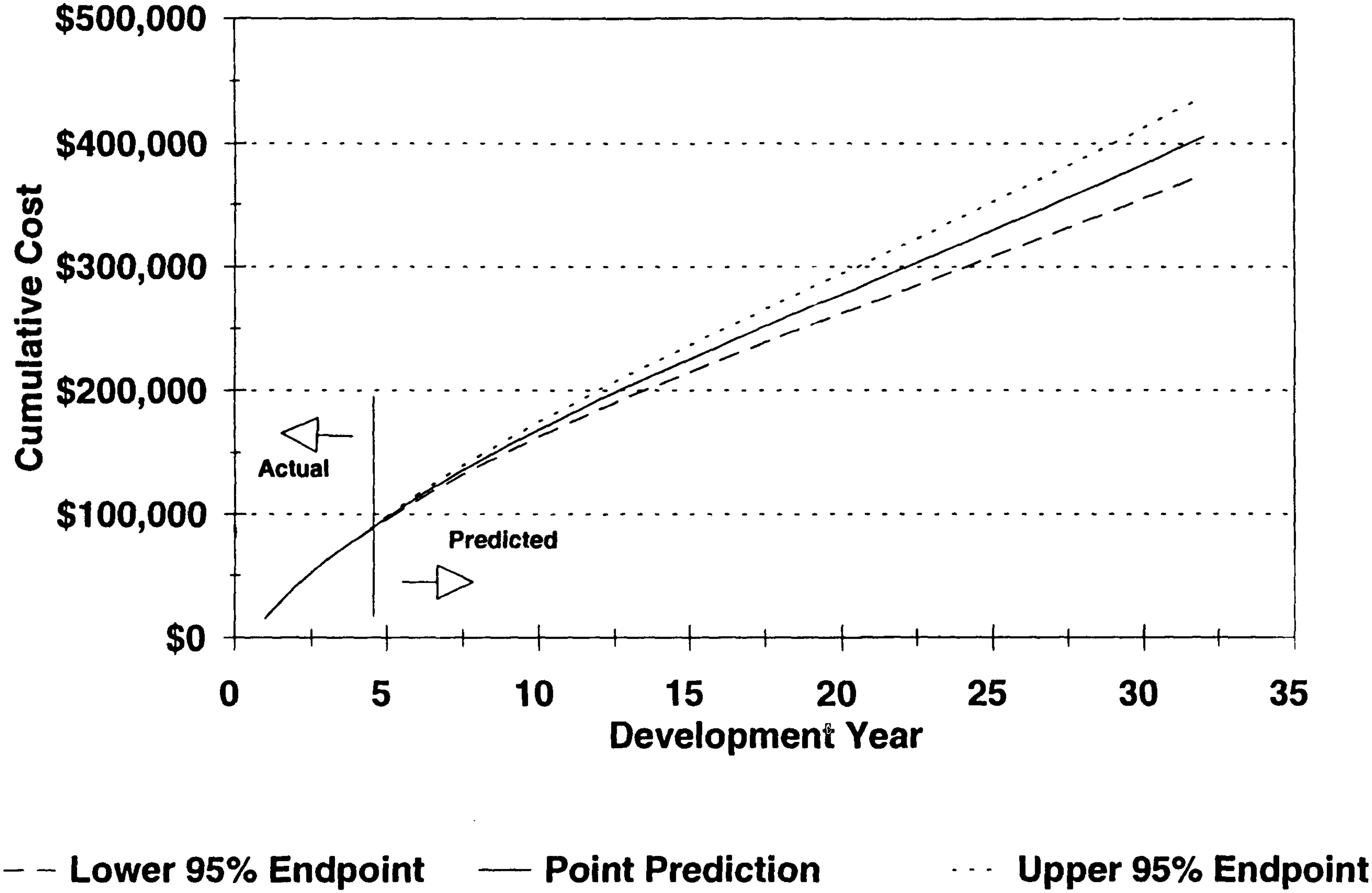


Table 3.3. Cumulative Loss Development Factors: Predicted and Prediction Interval Endpoints

Cohort Year 1989									
Year	0.0100	0.0250	0.0500	0.1000	Predicted	0.9000	0.9500	0.9750	0.9900
6	1.1485	1.1515	1.1541	1.1571	1.1678	1.1785	1.1816	1.1843	1.1874
7	1.2931	1.2980	1.3022	1.3070	1.3242	1.3416	1.3466	1.3510	1.3560
8	1.4286	1.4352	1.4409	1.4475	1.4710	1.4948	1.5017	1.5076	1.5146
9	1.5561	1.5644	1.5716	1.5799	1.6096	1.6399	1.6486	1.6562	1.6650
10	1.6768	1.6868	1.6955	1.7056	1.7416	1.7783	1.7889	1.7981	1.8088
11	1.7918	1.8036	1.8138	1.8256	1.8679	1.9112	1.9236	1.9345	1.9472
12	1.9022	1.9157	1.9274	1.9410	1.9897	2.0395	2.0539	2.0665	2.0811
13	2.0088	2.0240	2.0372	2.0526	2.1077	2.1643	2.1806	2.1948	2.2115
14	2.1121	2.1292	2.1439	2.1611	2.2227	2.2861	2.3044	2.3204	2.3391
15	2.2130	2.2318	2.2481	2.2671	2.3353	2.4056	2.4259	2.4437	2.4645
16	2.3118	2.3324	2.3503	2.3711	2.4461	2.5234	2.5457	2.5653	2.5882
17	2.4089	2.4314	2.4509	2.4736	2.5554	2.6398	2.6643	2.6857	2.7108
18	2.5048	2.5292	2.5503	2.5750	2.6637	2.7554	2.7820	2.8053	2.8326
19	2.5998	2.6261	2.6489	2.6755	2.7712	2.8705	2.8992	2.9244	2.9540
20	2.6942	2.7224	2.7469	2.7754	2.8784	2.9853	3.0163	3.0434	3.0753
21	2.7881	2.8183	2.8445	2.8751	2.9855	3.1001	3.1334	3.1625	3.1968
22	2.8819	2.9141	2.9421	2.9747	3.0926	3.2[illegible]	3.2508	3.2821	3.3187
23	2.9757	3.0099	3.0397	3.0744	3.2001	3.3308	3.3689	3.4022	3.4414
24	3.0697	3.1060	3.1376	3.1745	3.3080	3.4472	3.4877	3.5232	3.5649
25	3.1640	3.2025	3.2360	3.2751	3.4167	3.5644	3.6074	3.6451	3.6895
26	3.2588	3.2995	3.3349	3.3762	3.5261	3.6826	3.7283	3.7683	3.8154
27	3.3542	3.3972	3.4346	3.4782	3.6365	3.8021	3.8504	3.8928	3.9426
28	3.4504	3.4956	3.5350	3.5810	3.7481	3.9229	3.9740	4.0188	4.0715
29	3.5474	3.5950	3.6365	3.6849	3.8609	4.0452	4.0991	4.1464	4.2020
30	3.6453	3.6954	3.7389	3.7898	3.9750	4.1692	4.2259	4.2758	4.3344
31	3.7443	3.7968	3.8426	3.8960	4.0906	4.2948	4.3546	4.4070	4.4688
32	3.8445	3.8995	3.9475	4.0036	4.2077	4.4223	4.4851	4.5403	4.6054

Cohort Year 1990									
Year	0.0100	0.0250	0.0500	0.1000	Predicted	0.9000	0.9500	0.9750	0.9900
5	1.1974	1.2006	1.2033	1.2065	1.2177	1.2290	1.2322	1.2350	1.2383
6	1.3856	1.3909	1.3954	1.4007	1.4193	1.4382	1.4436	1.4483	1.4538
7	1.5603	1.5676	1.5739	1.5812	1.6072	1.6336	1.6412	1.6478	1.6555
8	1.7232	1.7326	1.7406	1.7500	1.7834	1.8175	1.8272	1.8358	1.8457
9	1.8763	1.8877	1.8975	1.9090	1.9499	1.9917	2.0037	2.0141	2.0264
10	2.0211	2.0345	2.0462	2.0597	2.1082	2.1578	2.1721	2.1846	2.1991
11	2.1590	2.1746	2.1881	2.2037	2.2599	2.3174	2.3340	2.3484	2.3654
12	2.2914	2.3091	2.3244	2.3422	2.4060	2.4716	2.4905	2.5070	2.5263
13	2.4192	2.4389	2.4561	2.4760	2.5477	2.6214	2.6426	2.6612	2.6830
14	2.5431	2.5651	2.5841	2.6062	2.6857	2.7677	2.7914	2.8121	2.8363
15	2.6640	2.6882	2.7091	2.7334	2.8209	2.9113	2.9374	2.9603	2.9870
16	2.7825	2.8088	2.8316	2.8582	2.9539	3.0527	3.0814	3.1064	3.1358
17	2.8991	2.9276	2.9524	2.9812	3.0851	3.1927	3.2239	3.2511	3.2831
18	3.0141	3.0449	3.0717	3.1028	3.2151	3.3316	3.3653	3.3949	3.4296
19	3.1281	3.1612	3.1899	3.2234	3.3443	3.4698	3.5062	3.5381	3.5755
20	3.2413	3.2767	3.3075	3.3434	3.4731	3.6078	3.6469	3.6812	3.7214
21	3.3540	3.3919	3.4247	3.4631	3.6016	3.7458	3.7877	3.8244	3.8675
22	3.4666	3.5068	3.5418	3.5826	3.7304	3.8842	3.9289	3.9681	4.0142
23	3.5791	3.6219	3.6591	3.7024	3.8595	4.0232	4.0708	4.1126	4.1617
24	3.6920	3.7372	3.7766	3.8226	3.9892	4.1630	4.2136	4.2581	4.3103
25	3.8052	3.8531	3.8947	3.9433	4.1197	4.3039	4.3576	4.4047	4.4602
26	3.9190	3.9696	4.0135	4.0649	4.2512	4.4461	4.5030	4.5529	4.6116
27	4.0336	4.0868	4.1332	4.1873	4.3839	4.5898	4.6498	4.7026	4.7647
28	4.1491	4.2051	4.2539	4.3109	4.5180	4.7350	4.7985	4.8541	4.9197
29	4.2656	4.3244	4.3757	4.4356	4.6535	4.8821	4.9490	5.0077	5.0768
30	4.3832	4.4450	4.4988	4.5617	4.7907	5.0312	5.1015	5.1633	5.2361
31	4.5021	4.5669	4.6233	4.6893	4.9297	5.1823	5.2562	5.3212	5.3978
32	4.6224	4.6902	4.7494	4.8185	5.0705	5.3357	5.4133	5.4816	5.5620

Table 3.3. Cumulative Loss Development Factors: Predicted and Prediction Interval Endpoints

Cohort Year 1991									
Year	0.0100	0.0250	0.0500	0.1000	Predicted	0.9000	0.9500	0.9750	0.9900
4	1.2829	1.2864	1.2893	1.2928	1.3049	1.3172	1.3207	1.3237	1.3273
5	1.5463	1.5523	1.5574	1.5634	1.5846	1.6061	1.6122	1.6176	1.6238
6	1.7884	1.7970	1.8044	1.8129	1.8434	1.8744	1.8833	1.8910	1.9000
7	2.0124	2.0236	2.0332	2.0444	2.0844	2.1252	2.1369	2.1471	2.1590
8	2.2210	2.2349	2.2468	2.2607	2.3104	2.3612	2.3757	2.3885	2.4034
9	2.4169	2.4334	2.4477	2.4643	2.5238	2.5847	2.6022	2.6175	2.6354
10	2.6022	2.6214	2.6381	2.6574	2.7268	2.7980	2.8185	2.8364	2.8573
11	2.7788	2.8007	2.8198	2.8419	2.9212	3.0027	3.0263	3.0468	3.0709
12	2.9482	2.9728	2.9943	3.0191	3.1085	3.2006	3.2272	3.2504	3.2777
13	3.1116	3.1391	3.1629	3.1906	3.2902	3.3929	3.4226	3.4485	3.4789
14	3.2703	3.3005	3.3268	3.3573	3.4672	3.5807	3.6135	3.6422	3.6759
15	3.4251	3.4581	3.4868	3.5202	3.6405	3.7650	3.8010	3.8325	3.8695
16	3.5767	3.6127	3.6438	3.6801	3.8110	3.9466	3.9859	4.0203	4.0606
17	3.7260	3.7648	3.7985	3.8377	3.9793	4.1262	4.1688	4.2062	4.2500
18	3.8733	3.9151	3.9513	3.9935	4.1461	4.3045	4.3505	4.3908	4.4381
19	4.0193	4.0640	4.1029	4.1481	4.3118	4.4820	4.5315	4.5748	4.6257
20	4.1643	4.2121	4.2536	4.3019	4.4770	4.6592	4.7122	4.7586	4.8132
21	4.3088	4.3596	4.4038	4.4553	4.6420	4.8364	4.8930	4.9426	5.0009
22	4.4530	4.5070	4.5539	4.6087	4.8071	5.0141	5.0744	5.1273	5.1895
23	4.5972	4.6544	4.7042	4.7622	4.9728	5.1927	5.2567	5.3129	5.3790
24	4.7418	4.8023	4.8549	4.9163	5.1393	5.3723	5.4402	5.4999	5.5700
25	4.8870	4.9508	5.0064	5.0712	5.3068	5.5533	5.6252	5.6884	5.7627
26	5.0329	5.1001	5.1587	5.2271	5.4756	5.7360	5.8120	5.8787	5.9573
27	5.1798	5.2505	5.3122	5.3842	5.6460	5.9205	6.0007	6.0712	6.1541
28	5.3278	5.4022	5.4670	5.5427	5.8181	6.1072	6.1917	6.2660	6.3534
29	5.4772	5.5552	5.6233	5.7027	5.9921	6.2962	6.3851	6.4633	6.5554
30	5.6281	5.7099	5.7812	5.8645	6.1682	6.4877	6.5812	6.6634	6.7602
31	5.7806	5.8662	5.9409	6.0283	6.3466	6.6819	6.7801	6.8664	6.9682
32	5.9348	6.0244	6.1026	6.1940	6.5275	6.8789	6.9820	7.0726	7.1794

Cohort Year 1992									
Year	0.0100	0.0250	0.0500	0.1000	Predicted	0.9000	0.9500	0.9750	0.9900
3	1.4845	1.4886	1.4921	1.4962	1.5106	1.5252	1.5294	1.5330	1.5372
4	1.9137	1.9213	1.9279	1.9355	1.9626	1.9901	1.9980	2.0049	2.0129
5	2.3032	2.3146	2.3244	2.3358	2.3765	2.4179	2.4298	2.4401	2.4522
6	2.6604	2.6757	2.6889	2.7043	2.7591	2.8151	2.8312	2.8452	2.8616
7	2.9904	3.0098	3.0265	3.0459	3.1152	3.1862	3.2066	3.2244	3.2453
8	3.2978	3.3212	3.3414	3.3649	3.4490	3.5353	3.5601	3.5818	3.6072
9	3.5863	3.6137	3.6375	3.6651	3.7642	3.8659	3.8953	3.9209	3.9509
10	3.8592	3.8907	3.9181	3.9498	4.0639	4.1813	4.2152	4.2448	4.2795
11	4.1192	4.1549	4.1858	4.2217	4.3510	4.4842	4.5226	4.5563	4.5957
12	4.3687	4.4085	4.4430	4.4831	4.6276	4.7768	4.8199	4.8576	4.9019
13	4.6095	4.6534	4.6916	4.7359	4.8958	5.0611	5.1090	5.1508	5.2000
14	4.8432	4.8914	4.9332	4.9818	5.1572	5.3389	5.3915	5.4376	5.4916
15	5.0713	5.1237	5.1692	5.2221	5.4133	5.6115	5.6689	5.7193	5.7783
16	5.2948	5.3515	5.4007	5.4580	5.6651	5.8801	5.9425	5.9972	6.0614
17	5.5147	5.5757	5.6287	5.6905	5.9138	6.1459	6.2133	6.2724	6.3418
18	5.7319	5.7973	5.8542	5.9204	6.1602	6.4097	6.4822	6.5458	6.6205
19	5.9471	6.0170	6.0778	6.1486	6.4051	6.6723	6.7501	6.8182	6.8983
20	6.1610	6.2354	6.3001	6.3756	6.6492	6.9345	7.0175	7.0904	7.1761
21	6.3740	6.4530	6.5218	6.6020	6.8930	7.1968	7.2853	7.3630	7.4543
22	6.5867	6.6704	6.7433	6.8284	7.1371	7.4598	7.5539	7.6365	7.7336
23	6.7994	6.8880	6.9652	7.0552	7.3820	7.7240	7.8238	7.9115	8.0145
24	7.0127	7.1062	7.1877	7.2827	7.6281	7.9899	8.0956	8.1884	8.2975
25	7.2269	7.3254	7.4112	7.5114	7.8758	8.2579	8.3696	8.4676	8.5831
26	7.4422	7.5458	7.6362	7.7416	8.1255	8.5284	8.6462	8.7497	8.8715
27	7.6590	7.7679	7.8628	7.9736	8.3774	8.8016	8.9257	9.0348	9.1633
28	7.8774	7.9917	8.0913	8.2077	8.6319	9.0780	9.2086	9.3234	9.4587
29	8.0979	8.2177	8.3221	8.4442	8.8893	9.3579	9.4951	9.6158	9.7580
30	8.3206	8.4460	8.5554	8.6833	9.1498	9.6415	9.7856	9.9123	10.0617
31	8.5457	8.6769	8.7913	8.9251	9.4137	9.9291	10.0802	10.2132	10.3700
32	8.7734	8.9105	9.0301	9.1701	9.6813	10.2210	10.3794	10.5187	10.6831

Table 3.3. Cumulative Loss Development Factors: Predicted and Prediction Interval Endpoints

*Cohort Year 1993									
Year					**Actual				
1					1.0000				
	0.0100	0.0250	0.0500	0.1000	Predicted	0.9000	0.9500	0.9750	0.9900
2	2.7918	2.8001	2.8073	2.8155	2.8450	2.8747	2.8831	2.8905	2.8991
3	4.1536	4.1708	4.1857	4.2030	4.2645	4.3268	4.3447	4.3602	4.3783
4	5.3407	5.3679	5.3914	5.4186	5.5157	5.6145	5.6428	5.6675	5.6964
5	6.4157	6.4534	6.4861	6.5239	6.6592	6.7973	6.8370	6.8716	6.9120
6	7.4004	7.4491	7.4913	7.5402	7.7154	7.8946	7.9461	7.9911	8.0437
7	8.3098	8.3697	8.4216	8.4819	8.6978	8.9193	8.9831	9.0388	9.1040
8	9.1563	9.2276	9.2893	9.3610	9.6184	9.8828	9.9591	10.0257	10.1037
9	9.9508	10.0334	10.1051	10.1883	10.4874	10.7953	10.8842	10.9619	11.0529
10	10.7022	10.7963	10.8779	10.9727	11.3138	11.6656	11.7672	11.8562	11.9604
11	11.4182	11.5238	11.6153	11.7218	12.1052	12.5012	12.6158	12.7160	12.8335
12	12.1052	12.2222	12.3238	12.4420	12.8680	13.3086	13.4363	13.5479	13.6789
13	12.7684	12.8970	13.0087	13.1387	13.6076	14.0933	14.2341	14.3574	14.5020
14	13.4122	13.5525	13.6744	13.8162	14.3286	14.8599	15.0140	15.1491	15.3075
15	14.0404	14.1925	14.3247	14.4785	15.0347	15.6123	15.7800	15.9269	16.0994
16	14.6562	14.8202	14.9628	15.1288	15.7294	16.3539	16.5353	16.6944	16.8812
17	15.2623	15.4383	15.5914	15.7698	16.4154	17.0876	17.2831	17.4544	17.6558
18	15.8609	16.0492	16.2129	16.4038	17.0953	17.8159	18.0257	18.2096	18.4258
19	16.4540	16.6548	16.8294	17.0330	17.7710	18.5411	18.7654	18.9622	19.1935
20	17.0435	17.2569	17.4425	17.6590	18.4446	19.2651	19.5043	19.7141	19.9609
21	17.6308	17.8570	18.0539	18.2836	19.1176	19.9896	20.2439	20.4671	20.7297
22	18.2173	18.4566	18.6650	18.9081	19.7915	20.7161	20.9860	21.2229	21.5017
23	18.8041	19.0568	19.2769	19.5337	20.4676	21.4460	21.7318	21.9828	22.2781
24	19.3924	19.6587	19.8907	20.1616	21.1470	22.1806	22.4827	22.7480	23.0604
25	19.9831	20.2633	20.5076	20.7928	21.8310	22.9210	23.2398	23.5199	23.8497
26	20.5771	20.8716	21.1282	21.4281	22.5204	23.6683	24.0042	24.2994	24.6472
27	21.1751	21.4842	21.7536	22.0685	23.2161	24.4234	24.7769	25.0877	25.4538
28	21.7780	22.1019	22.3845	22.7147	23.9190	25.1872	25.5588	25.8855	26.2706
29	22.3863	22.7256	23.0215	23.3674	24.6299	25.9606	26.3508	26.6939	27.0984
30	23.0008	23.3557	23.6653	24.0274	25.3495	26.7444	27.1536	27.5136	27.9381
31	23.6220	23.9929	24.3166	24.6952	26.0785	27.5394	27.9682	28.3455	28.7905
32	24.2505	24.6378	24.9759	25.3715	26.8176	28.3462	28.7951	29.1903	29.6564

* These apply to the 1993 cohort only. While numbers for cohort years in the near future will be similar, caution should be used in basing predictions for cohorts in the distant future on these for the 1993 cohort.

** This can be interpreted as a $1.00 cost in year 1. For example, an initial cost of $1.00 in 1993 will grow to a total accumulated cost of $26.82 by year 32.

Table 3.4. Loss Development Factor: Predicted and Prediction Interval Endpoints

Cohort Year 1989									
Year	0.0100	0.0250	0.0500	0.1000	Predicted	0.9000	0.9500	0.9750	0.9900
6	1.1485	1.1515	1.1541	1.1571	1.1678	1.1785	1.1816	1.1843	1.1874
7	1.1153	1.1182	1.1207	1.1236	1.1340	1.1444	1.1474	1.1499	1.1530
8	1.0926	1.0954	1.0979	1.1007	1.1108	1.1210	1.1239	1.1265	1.1294
9	1.0763	1.0791	1.0815	1.0843	1.0943	1.1043	1.1072	1.1096	1.1125
10	1.0642	1.0670	1.0694	1.0721	1.0820	1.0919	1.0947	1.0971	1.1000
11	1.0550	1.0577	1.0601	1.0628	1.0725	1.0824	1.0852	1.0876	1.0904
12	1.0477	1.0504	1.0528	1.0555	1.0652	1.0749	1.0777	1.0801	1.0829
13	1.0420	1.0447	1.0470	1.0497	1.0593	1.0690	1.0718	1.0742	1.0770
14	1.0373	1.0400	1.0423	1.0450	1.0546	1.0642	1.0670	1.0694	1.0722
15	1.0334	1.0361	1.0385	1.0411	1.0507	1.0603	1.0630	1.0654	1.0682
16	1.0302	1.0329	1.0352	1.0379	1.0474	1.0570	1.0597	1.0621	1.0649
17	1.0276	1.0302	1.0325	1.0352	1.0447	1.0542	1.0570	1.0593	1.0621
18	1.0253	1.0280	1.0303	1.0329	1.0424	1.0519	1.0546	1.0570	1.0598
19	1.0233	1.0260	1.0283	1.0310	1.0404	1.0499	1.0526	1.0550	1.0577
20	1.0216	1.0243	1.0266	1.0293	1.0387	1.0482	1.0509	1.0532	1.0560
21	1.0202	1.0228	1.0251	1.0278	1.0372	1.0467	1.0494	1.0517	1.0545
22	1.0189	1.0216	1.0238	1.0265	1.0359	1.0454	1.0481	1.0504	1.0532
23	1.0178	1.0204	1.0227	1.0254	1.0347	1.0442	1.0469	1.0493	1.0520
24	1.0168	1.0194	1.0217	1.0244	1.0337	1.0432	1.0459	1.0482	1.0510
25	1.0159	1.0185	1.0208	1.0235	1.0328	1.0423	1.0450	1.0473	1.0501
26	1.0151	1.0178	1.0200	1.0227	1.0320	1.0415	1.0442	1.0465	1.0493
27	1.0144	1.0170	1.0193	1.0220	1.0313	1.0408	1.0435	1.0458	1.0485
28	1.0138	1.0164	1.0187	1.0213	1.0307	1.0401	1.0428	1.0451	1.0479
29	1.0132	1.0158	1.0181	1.0207	1.0301	1.0395	1.0422	1.0445	1.0473
30	1.0127	1.0153	1.0176	1.0202	1.0296	1.0390	1.0417	1.0440	1.0467
31	1.0122	1.0148	1.0171	1.0197	1.0291	1.0385	1.0412	1.0435	1.0463
32	1.0118	1.0144	1.0167	1.0193	1.0286	1.0381	1.0407	1.0431	1.0458

Cohort Year 1990									
Year	0.0100	0.0250	0.0500	0.1000	Predicted	0.9000	0.9500	0.9750	0.9900
5	1.1974	1.2006	1.2033	1.2065	1.2177	1.2290	1.2322	1.2350	1.2383
6	1.1463	1.1493	1.1519	1.1549	1.1656	1.1764	1.1794	1.1821	1.1852
7	1.1137	1.1166	1.1191	1.1220	1.1324	1.1428	1.1458	1.1484	1.1514
8	1.0914	1.0942	1.0967	1.0995	1.1096	1.1198	1.1227	1.1253	1.1282
9	1.0754	1.0782	1.0806	1.0834	1.0933	1.1034	1.1062	1.1087	1.1116
10	1.0635	1.0662	1.0686	1.0714	1.0812	1.0911	1.0939	1.0964	1.0993
11	1.0543	1.0571	1.0595	1.0622	1.0719	1.0817	1.0845	1.0870	1.0898
12	1.0472	1.0499	1.0523	1.0550	1.0647	1.0744	1.0772	1.0796	1.0824
13	1.0415	1.0442	1.0466	1.0493	1.0589	1.0686	1.0713	1.0737	1.0765
14	1.0369	1.0396	1.0419	1.0446	1.0542	1.0638	1.0666	1.0690	1.0718
15	1.0331	1.0358	1.0381	1.0408	1.0503	1.0599	1.0627	1.0651	1.0678
16	1.0300	1.0326	1.0350	1.0376	1.0471	1.0567	1.0594	1.0618	1.0646
17	1.0273	1.0300	1.0323	1.0350	1.0444	1.0540	1.0567	1.0591	1.0618
18	1.0251	1.0277	1.0300	1.0327	1.0421	1.0517	1.0544	1.0568	1.0595
19	1.0231	1.0258	1.0281	1.0307	1.0402	1.0497	1.0524	1.0548	1.0575
20	1.0215	1.0241	1.0264	1.0291	1.0385	1.0480	1.0507	1.0531	1.0558
21	1.0200	1.0227	1.0250	1.0276	1.0370	1.0465	1.0492	1.0516	1.0543
22	1.0187	1.0214	1.0237	1.0263	1.0357	1.0452	1.0479	1.0503	1.0530
23	1.0176	1.0203	1.0226	1.0252	1.0346	1.0441	1.0468	1.0491	1.0519
24	1.0167	1.0193	1.0216	1.0242	1.0336	1.0431	1.0458	1.0481	1.0508
25	1.0158	1.0184	1.0207	1.0234	1.0327	1.0422	1.0449	1.0472	1.0499
26	1.0150	1.0176	1.0199	1.0226	1.0319	1.0414	1.0441	1.0464	1.0491
27	1.0143	1.0169	1.0192	1.0219	1.0312	1.0407	1.0434	1.0457	1.0484
28	1.0137	1.0163	1.0186	1.0212	1.0306	1.0400	1.0427	1.0450	1.0478
29	1.0131	1.0157	1.0180	1.0207	1.0300	1.0394	1.0421	1.0445	1.0472
30	1.0126	1.0152	1.0175	1.0201	1.0295	1.0389	1.0416	1.0439	1.0467
31	1.0121	1.0148	1.0170	1.0197	1.0290	1.0384	1.0411	1.0435	1.0462
32	1.0117	1.0143	1.0166	1.0192	1.0286	1.0380	1.0407	1.0430	1.0457

Table 3.4. Loss Development Factor: Predicted and Prediction Interval Endpoints

Cohort Year 1991									
Year	0.0100	0.0250	0.0500	0.1000	Predicted	0.9000	0.9500	0.9750	0.9900
4	1.2829	1.2864	1.2893	1.2928	1.3049	1.3172	1.3207	1.3237	1.3273
5	1.1941	1.1972	1.2000	1.2031	1.2143	1.2256	1.2289	1.2317	1.2349
6	1.1440	1.1470	1.1496	1.1527	1.1633	1.1741	1.1772	1.1798	1.1829
7	1.1121	1.1150	1.1175	1.1204	1.1307	1.1412	1.1441	1.1467	1.1497
8	1.0902	1.0930	1.0955	1.0983	1.1084	1.1186	1.1215	1.1240	1.1270
9	1.0744	1.0772	1.0796	1.0824	1.0924	1.1024	1.1053	1.1077	1.1106
10	1.0627	1.0655	1.0679	1.0706	1.0804	1.0903	1.0932	1.0956	1.0985
11	1.0537	1.0565	1.0588	1.0616	1.0713	1.0811	1.0839	1.0863	1.0892
12	1.0467	1.0494	1.0518	1.0545	1.0641	1.0739	1.0767	1.0791	1.0819
13	1.0411	1.0438	1.0461	1.0488	1.0584	1.0681	1.0709	1.0733	1.0761
14	1.0365	1.0392	1.0416	1.0442	1.0538	1.0634	1.0662	1.0686	1.0714
15	1.0328	1.0355	1.0378	1.0405	1.0500	1.0596	1.0623	1.0647	1.0675
16	1.0297	1.0323	1.0347	1.0373	1.0468	1.0564	1.0591	1.0615	1.0643
17	1.0270	1.0297	1.0320	1.0347	1.0442	1.0537	1.0564	1.0588	1.0616
18	1.0248	1.0275	1.0298	1.0325	1.0419	1.0514	1.0542	1.0565	1.0593
19	1.0229	1.0256	1.0279	1.0305	1.0400	1.0495	1.0522	1.0546	1.0573
20	1.0213	1.0239	1.0262	1.0289	1.0383	1.0478	1.0505	1.0529	1.0556
21	1.0198	1.0225	1.0248	1.0274	1.0368	1.0463	1.0490	1.0514	1.0541
22	1.0186	1.0212	1.0235	1.0262	1.0356	1.0451	1.0478	1.0501	1.0529
23	1.0175	1.0201	1.0224	1.0251	1.0345	1.0439	1.0466	1.0490	1.0517
24	1.0165	1.0192	1.0215	1.0241	1.0335	1.0429	1.0456	1.0480	1.0507
25	1.0157	1.0183	1.0206	1.0232	1.0326	1.0421	1.0447	1.0471	1.0498
26	1.0149	1.0175	1.0198	1.0225	1.0318	1.0413	1.0440	1.0463	1.0490
27	1.0142	1.0168	1.0191	1.0218	1.0311	1.0406	1.0432	1.0456	1.0483
28	1.0136	1.0162	1.0185	1.0211	1.0305	1.0399	1.0426	1.0449	1.0477
29	1.0130	1.0157	1.0179	1.0206	1.0299	1.0393	1.0420	1.0444	1.0471
30	1.0125	1.0151	1.0174	1.0201	1.0294	1.0388	1.0415	1.0438	1.0466
31	1.0120	1.0147	1.0170	1.0196	1.0289	1.0383	1.0410	1.0434	1.0461
32	1.0116	1.0143	1.0165	1.0192	1.0285	1.0379	1.0406	1.0429	1.0457

Cohort Year 1992									
Year	0.0100	0.0250	0.0500	0.1000	Predicted	0.9000	0.9500	0.9750	0.9900
3	1.4845	1.4886	1.4921	1.4962	1.5106	1.5252	1.5294	1.5330	1.5372
4	1.2772	1.2807	1.2836	1.2871	1.2992	1.3115	1.3150	1.3181	1.3216
5	1.1906	1.1938	1.1965	1.1997	1.2109	1.2222	1.2254	1.2282	1.2315
6	1.1417	1.1447	1.1473	1.1503	1.1610	1.1718	1.1748	1.1775	1.1806
7	1.1104	1.1133	1.1158	1.1187	1.1291	1.1395	1.1425	1.1450	1.1481
8	1.0889	1.0917	1.0942	1.0971	1.1071	1.1173	1.1202	1.1228	1.1257
9	1.0734	1.0762	1.0786	1.0814	1.0914	1.1014	1.1043	1.1067	1.1096
10	1.0619	1.0647	1.0671	1.0698	1.0796	1.0895	1.0923	1.0948	1.0977
11	1.0531	1.0558	1.0582	1.0609	1.0706	1.0804	1.0832	1.0857	1.0885
12	1.0461	1.0489	1.0512	1.0539	1.0636	1.0733	1.0761	1.0785	1.0813
13	1.0406	1.0433	1.0457	1.0484	1.0580	1.0676	1.0704	1.0728	1.0756
14	1.0361	1.0388	1.0411	1.0438	1.0534	1.0630	1.0658	1.0682	1.0710
15	1.0324	1.0351	1.0374	1.0401	1.0496	1.0593	1.0620	1.0644	1.0671
16	1.0294	1.0320	1.0344	1.0370	1.0465	1.0561	1.0588	1.0612	1.0640
17	1.0268	1.0295	1.0318	1.0344	1.0439	1.0535	1.0562	1.0585	1.0613
18	1.0246	1.0273	1.0296	1.0322	1.0417	1.0512	1.0539	1.0563	1.0590
19	1.0227	1.0254	1.0277	1.0303	1.0398	1.0493	1.0520	1.0543	1.0571
20	1.0211	1.0237	1.0260	1.0287	1.0381	1.0476	1.0503	1.0527	1.0554
21	1.0197	1.0223	1.0246	1.0273	1.0367	1.0462	1.0489	1.0512	1.0540
22	1.0184	1.0211	1.0234	1.0260	1.0354	1.0449	1.0476	1.0499	1.0527
23	1.0173	1.0200	1.0223	1.0249	1.0343	1.0438	1.0465	1.0488	1.0516
24	1.0164	1.0190	1.0213	1.0240	1.0333	1.0428	1.0455	1.0478	1.0506
25	1.0155	1.0182	1.0205	1.0231	1.0325	1.0419	1.0446	1.0470	1.0497
26	1.0148	1.0174	1.0197	1.0223	1.0317	1.0411	1.0438	1.0462	1.0489
27	1.0141	1.0167	1.0190	1.0216	1.0310	1.0404	1.0431	1.0455	1.0482
28	1.0135	1.0161	1.0184	1.0210	1.0304	1.0398	1.0425	1.0448	1.0476
29	1.0129	1.0156	1.0178	1.0205	1.0298	1.0392	1.0419	1.0443	1.0470
30	1.0124	1.0151	1.0173	1.0200	1.0293	1.0387	1.0414	1.0438	1.0465
31	1.0120	1.0146	1.0169	1.0195	1.0288	1.0383	1.0410	1.0433	1.0460
32	1.0115	1.0142	1.0165	1.0191	1.0284	1.0378	1.0405	1.0429	1.0456

Table 3.4. Loss Development Factor: Predicted and Prediction Interval Endpoints

Cohort Year 1993									
Year	0.0100	0.0250	0.0500	0.1000	Predicted	0.9000	0.9500	0.9750	0.9900
2	2.7918	2.8001	2.8073	2.8155	2.8450	2.8747	2.8831	2.8905	2.8991
3	1.4728	1.4769	1.4804	1.4845	1.4990	1.5135	1.5177	1.5213	1.5255
4	1.2714	1.2748	1.2778	1.2812	1.2934	1.3057	1.3092	1.3122	1.3158
5	1.1871	1.1902	1.1930	1.1961	1.2073	1.2187	1.2219	1.2247	1.2280
6	1.1393	1.1423	1.1449	1.1479	1.1586	1.1694	1.1724	1.1751	1.1782
7	1.1087	1.1116	1.1141	1.1170	1.1273	1.1378	1.1407	1.1433	1.1463
8	1.0876	1.0904	1.0929	1.0957	1.1058	1.1160	1.1189	1.1214	1.1244
9	1.0724	1.0752	1.0776	1.0804	1.0904	1.1004	1.1032	1.1057	1.1086
10	1.0611	1.0638	1.0662	1.0690	1.0788	1.0887	1.0915	1.0940	1.0968
11	1.0524	1.0551	1.0575	1.0602	1.0700	1.0798	1.0826	1.0850	1.0878
12	1.0456	1.0483	1.0506	1.0534	1.0630	1.0728	1.0755	1.0779	1.0808
13	1.0401	1.0428	1.0452	1.0479	1.0575	1.0672	1.0699	1.0723	1.0751
14	1.0357	1.0384	1.0407	1.0434	1.0530	1.0626	1.0654	1.0678	1.0705
15	1.0321	1.0348	1.0371	1.0398	1.0493	1.0589	1.0616	1.0640	1.0668
16	1.0290	1.0317	1.0340	1.0367	1.0462	1.0558	1.0585	1.0609	1.0637
17	1.0265	1.0292	1.0315	1.0341	1.0436	1.0532	1.0559	1.0583	1.0610
18	1.0243	1.0270	1.0293	1.0320	1.0414	1.0509	1.0537	1.0560	1.0588
19	1.0225	1.0251	1.0274	1.0301	1.0395	1.0490	1.0518	1.0541	1.0569
20	1.0209	1.0235	1.0258	1.0285	1.0379	1.0474	1.0501	1.0525	1.0552
21	1.0195	1.0221	1.0244	1.0271	1.0365	1.0460	1.0487	1.0510	1.0538
22	1.0183	1.0209	1.0232	1.0259	1.0352	1.0447	1.0474	1.0498	1.0525
23	1.0172	1.0198	1.0221	1.0248	1.0342	1.0436	1.0463	1.0487	1.0514
24	1.0162	1.0189	1.0212	1.0238	1.0332	1.0427	1.0454	1.0477	1.0504
25	1.0154	1.0181	1.0203	1.0230	1.0323	1.0418	1.0445	1.0468	1.0496
26	1.0147	1.0173	1.0196	1.0222	1.0316	1.0410	1.0437	1.0461	1.0488
27	1.0140	1.0166	1.0189	1.0215	1.0309	1.0403	1.0430	1.0454	1.0481
28	1.0134	1.0160	1.0183	1.0209	1.0303	1.0397	1.0424	1.0447	1.0475
29	1.0128	1.0155	1.0177	1.0204	1.0297	1.0392	1.0418	1.0442	1.0469
30	1.0123	1.0150	1.0172	1.0199	1.0292	1.0386	1.0413	1.0437	1.0464
31	1.0119	1.0145	1.0168	1.0194	1.0288	1.0382	1.0409	1.0432	1.0459
32	1.0115	1.0141	1.0164	1.0190	1.0283	1.0378	1.0404	1.0428	1.0455

Table 3.5. Cumulative Costs: Predicted and Prediction Interval Endpoints (In Thousands of $)

Cohort Year 1989									
Year					Actual				
1					12,199				
2					37,324				
3					55,962				
4					72,550				
5					86,373				
	0.0100	0.0250	0.0500	0.1000	Predicted	0.9000	0.9500	0.9750	0.9900
6	99,196	99,456	99,681	99,941	100,864	101,794	102,060	102,291	102,560
7	111,691	112,109	112,470	112,888	114,374	115,880	116,311	116,686	117,123
8	123,394	123,963	124,454	125,023	127,052	129,113	129,704	130,218	130,818
9	134,404	135,122	135,742	136,462	139,029	141,645	142,395	143,049	143,813
10	144,829	145,696	146,446	147,315	150,423	153,597	154,509	155,304	156,234
11	154,767	155,783	156,663	157,683	161,336	165,074	166,149	167,087	168,184
12	164,300	165,467	166,478	167,650	171,853	176,161	177,402	178,486	179,753
13	173,502	174,821	175,963	177,289	182,048	186,934	188,343	189,574	191,015
14	182,432	183,903	185,179	186,660	191,982	197,456	199,036	200,416	202,033
15	191,141	192,767	194,178	195,817	201,709	207,779	209,533	211,066	212,862
16	199,673	201,456	203,003	204,802	211,274	217,951	219,882	221,570	223,550
17	208,065	210,008	211,693	213,654	220,716	228,011	230,123	231,970	234,136
18	216,349	218,454	220,281	222,406	230,068	237,994	240,290	242,299	244,656
19	224,554	226,823	228,793	231,086	239,360	247,930	250,414	252,589	255,142
20	232,703	235,140	237,256	239,720	248,617	257,845	260,523	262,867	265,620
21	240,818	243,425	245,691	248,329	257,863	267,764	270,639	273,157	276,115
22	248,918	251,699	254,116	256,932	267,118	277,707	280,784	283,481	286,649
23	257,018	259,977	262,550	265,548	276,399	287,694	290,979	293,858	297,241
24	265,135	268,276	271,007	274,191	285,723	297,741	301,239	304,306	307,910
25	273,282	276,608	279,502	282,875	295,106	307,865	311,581	314,841	318,673
26	281,471	284,987	288,046	291,615	304,560	318,080	322,020	325,477	329,543
27	289,713	293,423	296,652	300,420	314,098	328,399	332,570	336,230	340,536
28	298,018	301,927	305,331	309,303	323,732	338,834	343,242	347,112	351,665
29	306,397	310,510	314,092	318,273	333,473	349,399	354,050	358,135	362,942
30	314,857	319,179	322,944	327,340	343,331	360,103	365,004	369,310	374,379
31	323,408	327,943	331,896	336,512	353,315	370,957	376,116	380,649	385,987
32	332,057	336,812	340,956	345,798	363,435	381,971	387,395	392,162	397,778

Cohort Year 1990									
Year					Actual				
1					14,955				
2					41,424				
3					62,897				
4					79,971				
	0.0100	0.0250	0.0500	0.1000	Predicted	0.9000	0.9500	0.9750	0.9900
5	95,758	96,011	96,230	96,482	97,379	98,284	98,542	98,766	99,028
6	110,811	111,230	111,593	112,012	113,503	115,015	115,447	115,823	116,261
7	124,778	125,361	125,865	126,449	128,528	130,642	131,248	131,775	132,391
8	137,807	138,554	139,200	139,948	142,621	145,345	146,126	146,808	147,604
9	150,045	150,957	151,746	152,661	155,933	159,275	160,235	161,072	162,051
10	161,626	162,704	163,637	164,719	168,595	172,563	173,704	174,701	175,866
11	172,660	173,906	174,984	176,235	180,722	185,323	186,649	187,806	189,161
12	183,246	184,659	185,884	187,306	192,410	197,653	199,165	200,485	202,032
13	193,461	195,045	196,417	198,011	203,739	209,633	211,334	212,821	214,563
14	203,376	205,130	206,652	208,420	214,780	221,334	223,228	224,884	226,824
15	213,046	214,974	216,646	218,591	225,592	232,816	234,906	236,734	238,876
16	222,520	224,624	226,449	228,573	236,224	244,131	246,420	248,423	250,772
17	231,840	234,122	236,103	238,408	246,720	255,322	257,814	259,996	262,555
18	241,041	243,504	245,643	248,133	257,118	266,428	269,128	271,492	274,266
19	250,155	252,802	255,102	257,779	267,450	277,483	280,395	282,945	285,939
20	259,209	262,043	264,506	267,374	277,744	288,516	291,645	294,386	297,605
21	268,225	271,250	273,880	276,943	288,027	299,554	302,905	305,841	309,291
22	277,225	280,445	283,245	286,507	298,320	310,620	314,198	317,335	321,020
23	286,228	289,646	292,619	296,086	308,644	321,735	325,546	328,888	332,816
24	295,249	298,871	302,021	305,695	319,017	332,919	336,968	340,521	344,698
25	304,304	308,133	311,466	315,352	329,455	344,188	348,483	352,252	356,684
26	313,407	317,448	320,966	325,071	339,973	355,559	360,106	364,097	368,791
27	322,570	326,828	330,536	334,864	350,586	367,047	371,853	376,071	381,036
28	331,805	336,285	340,187	344,743	361,307	378,666	383,737	388,190	393,432
29	341,121	345,829	349,931	354,720	372,146	390,429	395,773	400,467	405,994
30	350,529	355,470	359,776	364,806	383,117	402,347	407,972	412,914	418,734
31	360,038	365,218	369,733	375,009	394,229	414,433	420,347	425,544	431,667
32	369,656	375,081	379,812	385,340	405,492	426,698	432,909	438,368	444,802

Table 3.5. Cumulative Costs: Predicted and Prediction Interval Endpoints (In Thousands of $)

Cohort Year 1991									
Year					Actual				
1					13,566				
2					40,314				
3					60,137				
	0.0100	0.0250	0.0500	0.1000	Predicted	0.9000	0.9500	0.9750	0.9900
4	77,151	77,358	77,536	77,742	78,473	79,211	79,421	79,604	79,817
5	92,990	93,349	93,659	94,017	95,292	96,585	96,954	97,276	97,651
6	107,551	108,065	108,509	109,023	110,856	112,720	113,254	113,719	114,263
7	121,018	121,690	122,271	122,945	125,350	127,802	128,506	129,119	129,836
8	133,566	134,398	135,118	135,953	138,940	141,992	142,870	143,635	144,530
9	145,345	146,339	147,200	148,198	151,774	155,436	156,490	157,410	158,487
10	156,489	157,646	158,647	159,810	163,981	168,260	169,494	170,571	171,831
11	167,107	168,427	169,571	170,900	175,671	180,575	181,990	183,226	184,674
12	177,293	178,778	180,065	181,561	186,938	192,474	194,073	195,470	197,108
13	187,124	188,775	190,207	191,871	197,860	204,036	205,822	207,383	209,213
14	196,665	198,484	200,062	201,897	208,505	215,330	217,305	219,032	221,058
15	205,973	207,962	209,687	211,694	218,930	226,413	228,580	230,477	232,701
16	215,094	217,254	219,129	221,311	229,183	237,335	239,698	241,766	244,194
17	224,068	226,402	228,428	230,787	239,306	248,138	250,701	252,945	255,579
18	232,930	235,439	237,620	240,159	249,334	258,861	261,627	264,050	266,895
19	241,708	244,397	246,734	249,456	259,301	269,535	272,508	275,114	278,175
20	250,429	253,300	255,796	258,705	269,233	280,188	283,374	286,167	289,448
21	259,116	262,173	264,831	267,930	279,154	290,847	294,251	297,234	300,741
22	267,788	271,034	273,858	277,151	289,086	301,535	305,160	308,340	312,078
23	276,463	279,903	282,896	286,387	299,048	312,270	316,123	319,504	323,479
24	285,158	288,795	291,961	295,654	309,059	323,072	327,159	330,745	334,964
25	293,887	297,725	301,068	304,968	319,134	333,958	338,284	342,082	346,550
26	302,662	306,707	310,230	314,341	329,287	344,943	349,515	353,529	358,254
27	311,496	315,752	319,459	323,788	339,532	356,042	360,866	365,103	370,091
28	320,399	324,871	328,768	333,319	349,881	367,267	372,351	376,817	382,076
29	329,383	334,075	338,166	342,944	360,347	378,632	383,982	388,683	394,221
30	338,455	343,374	347,663	352,675	370,939	390,148	395,772	400,716	406,540
31	347,625	352,777	357,270	362,521	381,668	401,827	407,732	412,925	419,044
32	356,902	362,292	366,993	372,489	392,544	413,678	419,874	425,323	431,745

Cohort Year 1992									
Year					Actual				
1					14,468				
2					41,982				
	0.0100	0.0250	0.0500	0.1000	Predicted	0.9000	0.9500	0.9750	0.9900
3	62,189	62,360	62,507	62,678	63,282	63,893	64,067	64,219	64,395
4	80,168	80,487	80,763	81,082	82,219	83,371	83,700	83,987	84,322
5	96,485	96,962	97,375	97,853	99,557	101,291	101,788	102,221	102,727
6	111,448	112,090	112,645	113,288	115,585	117,930	118,603	119,190	119,876
7	125,275	126,084	126,784	127,597	130,503	133,476	134,331	135,077	135,950
8	138,151	139,130	139,977	140,961	144,486	148,100	149,141	150,049	151,112
9	150,237	151,387	152,383	153,539	157,689	161,951	163,180	164,254	165,511
10	161,669	162,990	164,136	165,466	170,246	175,163	176,583	177,824	179,277
11	172,562	174,056	175,351	176,856	182,270	187,850	189,462	190,872	192,524
12	183,013	184,679	186,125	187,807	193,859	200,108	201,915	203,496	205,349
13	193,100	194,941	196,539	198,398	205,095	212,019	214,024	215,778	217,836
14	202,892	204,909	206,660	208,698	216,047	223,655	225,860	227,790	230,055
15	212,446	214,640	216,546	218,764	226,773	235,074	237,483	239,591	242,066
16	221,809	224,183	226,245	228,646	237,323	246,329	248,944	251,234	253,922
17	231,022	233,578	235,799	238,386	247,741	257,463	260,287	262,762	265,669
18	240,121	242,861	245,243	248,019	258,063	268,514	271,552	274,216	277,345
19	249,136	252,064	254,609	257,576	268,322	279,516	282,773	285,629	288,985
20	258,094	261,212	263,924	267,086	278,546	290,498	293,978	297,031	300,619
21	267,018	270,329	273,211	276,572	288,761	301,487	305,195	308,449	312,274
22	275,927	279,437	282,492	286,055	298,988	312,506	316,447	319,907	323,975
23	284,842	288,553	291,784	295,554	309,247	323,575	327,756	331,426	335,744
24	293,777	297,694	301,105	305,087	319,557	334,714	339,140	343,026	347,600
25	302,748	306,875	310,470	314,668	329,934	345,941	350,618	354,726	359,561
26	311,767	316,110	319,893	324,312	340,392	357,270	362,205	366,540	371,645
27	320,848	325,410	329,387	334,031	350,946	368,717	373,917	378,486	383,867
28	330,001	334,788	338,962	343,838	361,608	380,296	385,767	390,576	396,242
29	339,237	344,255	348,630	353,744	372,390	392,020	397,770	402,826	408,784
30	348,565	353,819	358,401	363,758	383,304	403,900	409,937	415,246	421,505
31	357,995	363,490	368,285	373,891	394,360	415,948	422,280	427,850	434,418
32	367,535	373,278	378,290	384,152	405,567	428,176	434,811	440,650	447,535

Table 3.5. Cumulative Costs: Predicted and Prediction Interval Endpoints (In Thousands of $)

Cohort Year 1993									
Year					Actual				
1					13,702				
	0.0100	0.0250	0.0500	0.1000	Predicted	0.9000	0.9500	0.9750	0.9900
2	38,253	38,366	38,465	38,578	38,981	39,388	39,504	39,605	39,723
3	56,912	57,148	57,353	57,589	58,431	59,286	59,530	59,743	59,991
4	73,178	73,550	73,872	74,245	75,575	76,929	77,318	77,656	78,051
5	87,907	88,425	88,872	89,391	91,244	93,137	93,680	94,154	94,708
6	101,400	102,068	102,646	103,316	105,716	108,171	108,877	109,494	110,215
7	113,860	114,681	115,392	116,218	119,177	122,211	123,085	123,849	124,742
8	125,460	126,436	127,282	128,264	131,790	135,413	136,459	137,371	138,440
9	136,345	137,477	138,459	139,599	143,698	147,916	149,135	150,199	151,447
10	146,641	147,930	149,048	150,347	155,021	159,841	161,234	162,452	163,880
11	156,452	157,898	159,153	160,612	165,865	171,291	172,861	174,234	175,845
12	165,865	167,469	168,861	170,480	176,317	182,354	184,103	185,633	187,428
13	174,952	176,714	178,245	180,025	186,451	193,106	195,036	196,725	198,706
14	183,773	185,696	187,366	189,310	196,329	203,610	205,722	207,572	209,743
15	192,381	194,465	196,276	198,385	206,005	213,919	216,217	218,230	220,594
16	200,819	203,066	205,019	207,294	215,524	224,080	226,567	228,746	231,305
17	209,123	211,535	213,633	216,077	224,924	234,133	236,812	239,160	241,919
18	217,325	219,905	222,149	224,764	234,239	244,113	246,987	249,507	252,469
19	225,453	228,203	230,595	233,385	243,498	254,050	257,123	259,819	262,988
20	233,530	236,453	238,997	241,964	252,727	263,970	267,247	270,122	273,503
21	241,577	244,676	247,374	250,522	261,949	273,897	277,382	280,440	284,038
22	249,613	252,892	255,747	259,079	271,182	283,851	287,549	290,796	294,615
23	257,654	261,116	264,131	267,651	280,446	293,853	297,769	301,207	305,254
24	265,714	269,363	272,542	276,254	289,756	303,918	308,058	311,693	315,973
25	273,808	277,648	280,994	284,902	299,128	314,064	318,432	322,269	326,789
26	281,947	285,982	289,499	293,608	308,574	324,303	328,905	332,950	337,715
27	290,141	294,376	298,068	302,382	318,107	334,649	339,493	343,751	348,768
28	298,401	302,840	306,711	311,236	327,738	345,115	350,206	354,683	359,959
29	306,737	311,385	315,440	320,180	337,479	355,712	361,058	365,759	371,301
30	315,157	320,019	324,262	329,223	347,339	366,452	372,058	376,991	382,807
31	323,669	328,750	333,185	338,373	357,328	377,344	383,219	388,389	394,487
32	332,280	337,587	342,219	347,640	367,455	388,399	394,550	399,965	406,352

4. Conclusions and Recommendations

This investigation shows that the Loss Development Method commonly used by actuaries to predict workers' compensation costs can be cast in the context of intrinsically linear models and thereby made amenable to the theory of linear statistical models for the computation of point and interval predictions of future costs. These computations have been illustrated using actual and imputed U.S. Department of the Navy workers' compensation claims. In addition, the log-loss development regression model developed herein has been shown to produce point predictions that are nearly the same as those produced (with apparently much more effort) using traditional actuarial methods (Figures 3.1 and 3.2). In contrast to the actuarial approach, the regression approach is relatively easy to compute, with the computations involved being standard in many statistical computer packages, and it provides a means of assessing the accuracy of the resultant predictions. Moreover, beyond knowledge of basic linear statistical models and statistical analysis, specialized knowledge is not needed to apply them.

I emphasize here that the reader should not conclude that an inexperienced analyst armed with the linear model methods employed herein can somehow replace the traditional analyses and/or services of a professional actuary. Rather, the methods of this investigation should be evaluated further by actuarial scientists and practitioners and perhaps be adopted (with any necessary and suitable modifications) as another set of tools in the collection of numerical methods that have come into actuarial practice.

Mathematically and statistically speaking, a few paths for future research should be pursued. First, recall that the linear model formulated in section 2 assumes that the error variances are not all equal. In fact, they are assumed to approach zero as development year increases. This is a reasonable assumption and one that should be retained. However, the examples used to illustrate the methods all assumed homogeneity of variances. This approach was taken in section 3.1 because the data set was so small and in section 3.2 because the data (which contained a great deal of imputed data) was already very smooth and the errors very small. In practice, an effort should be made to assess the weights in (2.2) perhaps considering expansions of w_j that would lead to $w_j = O(1/j^{\nu})$ as $j \to \infty$ for some $\nu > 1$.

A crucial assumption that allows the development of closed-form expressions for the prediction interval endpoints is that of a Gaussian error distribution. Some investigation should be undertaken using more extensive data to test the Gaussian assumption. This was not pursued in this investigation because the example of section 3.1 had too few data points and the data from section 3.2 had too much imputed data.

In parallel with testing the Gaussian assumption, alternative methods (not based on the Gaussian assumption) of computing prediction intervals should be investigated. In particular, the models of section 2 are all amenable to bootstrapping, a computational method that allows the estimation of the relevant statistical quantities with only minimal assumptions. I am currently pursuing this in a separate investigation that will make use only of the actual U.S. Department of the Navy data (as opposed to the imputed data in the study by Miccolis[3]).

Finally, I recommend that models having different regression structures (perhaps nonlinear, and not intrinsically linear), as well as some classes of dynamic statistical models (for example $Y_t = g(Y_{t-1}, \varepsilon_t)$, $t = 1, 2, \ldots$) be investigated. The actuarial literature is currently focused on ad hoc (although effective and useful) curve graduation and point projection methods and is relatively short of models with sufficient structure to allow probabilistic assessment of prediction accuracy. This report provides a stimulus in these directions.

REFERENCES

1. Doyle, E.J., Shepherd, S.L., & LaFleur, B.J. (1993). *Costs for Department of the Navy Civilians due to Federal Employees' Compensation Act: How much does a case cost?* (Report No. 93-6). San Diego, CA: Naval Health Research Center.

2. National Council on Compensation Insurance. (1993). *Annual Statistical Bulletin.* Boca Raton: Author.

3. Miccolis, J.A. (1993). Correspondence to Dr. Edward J. Doyle, Jr. & Mr. Steve Shepherd, Re: NAVOSH Mishap Cost Reduction Model, 17 December 1993.

4. Angus, J.E. (1991). *Criteria for choosing the best neural network: Part I.* (Report No. 91-16). San Diego, CA: Naval Health Research Center.

5. Arnold, S.F. (1981). *The theory of linear models and multivariate analysis.* New York: John Wiley.

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
August 1994

3. REPORT TYPE AND DATE COVERED
Final 1 Jun - 3 Aug 94

4. TITLE AND SUBTITLE
INTRINSICALLY LINEAR LOSS DEVELOPMENT MODELS FOR WORKERS COMPENSATION COSTS: POINT AND INTERVAL PREDICTION METHODS

5. FUNDING NUMBERS
Program Element: 63706N
Work Unit Number:
M0096.004-6202

6. AUTHOR(S)
J. E. ANGUS

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Naval Health Research Center
P. O. Box 85122
San Diego, CA 92186-5122

8. PERFORMING ORGANIZATION
Report No.
94-13

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Naval Medical Research and Development Command
National Naval Medical Center
Building 1, Tower 2
Bethesda, MD 20889-5044

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

This report describes modeling methods that allow the computation of point predictions and prediction probability intervals for cumulative workers compensation costs. Underlying these models is the actuarial loss development factor method, a method that computes projected costs by utilizing ratios of known cumulative costs in consecutive years. While the relationship between cumulative loss development, cohort, and development year in these models is nonlinear, a transformation readers them in the form of standard linear statistical models, thus allowing the development of prediction probability intervals when the error structure is Gaussian. The modeling methods are illustrated using data collected from U.S. Department of the Navy workers compensation payments made from 1990 through 1993, including claim costs originating from 1961 through 1993.

14. SUBJECT TERMS
Statistical Models, Probability Intervals, Cost Projections, Forecasts, Insurance Claims, Actuarial Methods

15. NUMBER OF PAGES
45

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	Unlimited